

**SPATIAL COMPETITION
BETWEEN SHOPPING
CENTERS**

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Spatial competition between shopping centers^{*}

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Abstract. We study competition between two shopping centers (department stores or shopping malls) located at the extremes of a linear city. In contrast with the existing literature, we do not restrict consumers to make all their purchases at a single place. We obtain this condition as an equilibrium result. In the case of competition between a shopping mall and a department store, we find that the shops at the mall, taken together, obtain a lower profit than the department store. However, the shops at the mall have no incentives to merge into a department store (both sides would lose). It is the department store that has incentives to separate itself into a shopping mall (both sides win).

Keywords: Retail organization, Multi-product firms, Horizontal differentiation, Hotelling model.

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1 Introduction

Shopping centers have existed for many centuries as galleries, market squares, bazaars or seaport districts. The oldest indoor space where consumers can buy a huge variety of goods is the *Al-Hamidiyah Souq*, in Damascus (Syria), and dates back to the seventh century.

One of the reasons why shopping centers are so attractive is because they allow consumers to buy many different kinds of goods without spending much time and money commuting between shops. Therefore, to study competition between shopping centers, one should take into account the demand for many different goods and also the commuting costs of traveling to one or more shopping centers. The existing spatial competition models fail to do so, because they either restrict the analysis to markets with a single good or assume that consumers make all their purchases at the same place (Bliss, 1988; Beggs, 1994; Smith and Hay, 2005; Innes, 2006). This “one stop shopping” assumption is very convenient because it allows treating multiple goods as a single bundled good.

We provide a study of competition between shopping centers by extending the standard model of spatial competition (Hotelling, 1929; d’Aspremont, Gabszewicz and Thisse, 1979) to the case of multiple goods without using the “one stop shopping” assumption. This extension is straightforward in concept but technically difficult. We consider the existence of two shopping centers located at the extremes of a linear city, selling the same set of goods. In the model, a shopping center may be either a shopping mall (where each good is sold by an independent firm) or a department store (where a single firm sells all the goods). Consumers are uniformly spread across the linear city and buy exactly one unit of each good. They may travel to a shopping center and buy all the goods there or travel to both shopping centers and buy each good where it is cheaper.¹

We solve for the equilibrium prices, market shares and profits in three scenarios of retail organization: *(i)* competition between a department store and a shopping mall; *(ii)* competition between two department stores; *(iii)* competition between two shopping malls. Our first result is that, regardless of the scenario that is considered, no consumer travels to both extremes of the city. “One stop shopping” is obtained as a result. The equilibrium price differences across shopping centers are not sufficient to make it worthwhile for a

¹Consumers are assumed to be fully informed about the prices charged in each extreme of the city.

consumer to travel to both extremes of the city.²

In the case of competition between a department store and a shopping mall, we find that the department store sets lower prices and captures a greater demand than the shops at the mall. To understand why this occurs, observe that unrelated goods become complements when they are sold at the same location (and substitutes when they are sold at different extremes of the city). When a shop at the mall considers the possibility of decreasing its price, it only cares about the increase of its own demand and not about the increase of the demand of the other shops at the mall. In contrast, the department store internalizes this effect, and takes into account the fact that decreasing the price of one good also increases the demand for the other goods that are sold there.³ Despite charging lower prices, the department store obtains a higher profit than the shops at the mall taken together because it captures a sufficiently greater market share.

When competition is between two department stores, the equilibrium prices are lower. The price that each department store charges for the bundle of goods is actually equal to the price charged in the single-good model (independently of the number of goods). The two department stores obviously capture equal shares of the market and obtain equal profits. These are, unsurprisingly, lower than the profits obtained when competing against a shopping mall (because a department store competes more aggressively).

Finally, in the scenario of competition between two shopping malls, we arrive at the same equilibrium price (for each good) as in the single-good model. The shops behave as if consumers only bought one good. This is the scenario in which prices are higher. The explanation is the same as before: the shops at the mall set higher prices because they do not internalize the positive effect of a price decrease on the other shops at the same mall.

After finding the equilibrium in each of the three competitive scenarios, it is straightforward to analyze whether it is more profitable to have a department store offering many

²Even ruling out bundling strategies (we assume that the price of a bundle of goods is equal to the sum of the price of the individual goods), we find that the consumers that travel to a department store buy all the goods there. Therefore, allowing the department store to charge for the bundle a price that is lower than the sum of the prices of the individual goods would make no difference. For a careful analysis of the bundle pricing problem, see, for example, Hanson and Martin (1990).

³The asymmetry between the department store and the shopping mall becomes more pronounced as the number of goods increases. We find that when the number of goods tends to infinity, the market share of the department store converges to 100%.

products or several independent shops at a mall. To find out which retail organization is expected to appear endogenously, we solve for the equilibrium of the corresponding merger game. Mergers are typically carried out to increase market power or to obtain cost savings. The integration of independent shops at a mall could be seen as a conglomerate merger, since the products involved have neither horizontal nor vertical relationships. However, we must not forget that the products sold in the same shopping center are complements. Thus, the merger of shops at the same mall is a merger between firms that sell complementary goods.⁴

In our merger game, it is a dominant strategy to be organized as a shopping mall rather than as a department store. Both sides win whenever a department store separates into several independent shops. Therefore, the retail organization that is expected to appear in equilibrium is that of competition between two shopping malls. This result is not surprising because the greater is the number of department stores, the more competitive is the retail industry. As we have explained above, a department store has stronger incentives to charge lower prices than the independent shops at a mall. If the prices of the rival retailers remained the same, behaving as a department store would be profitable. However, it induces the rivals to lower their prices as well. This competitive effect dominates, leading to lower prices and profits for everyone. It is better to be organized as a shopping mall because, as explained by Innes (2006): “*a multi-product retailer can effectively pre-commit to higher prices by organizing itself as a mall of independent outlets*”.

The first result of this kind was presented by Edgeworth (1925), who found that it is better, for consumers, to have a single monopolist selling two complementary goods than to have two separate monopolists. More recently, Salant, Switzer and Reynolds (1983) also came up with a similar result, but in a model of Cournot competition. Using a framework that is closer to ours, Bertrand competition with linear demand, Beggs (1994) concluded that separating into several shops at a mall may be desirable or not. Depending on whether the degree of substitutability between the goods sold at the competing shopping centers is low or high, either two department stores or two shopping malls emerge as equilibria of the merger game. Innes (2006) studied the effect of entry and concluded that only department stores survive in equilibrium because they compete more aggressively and,

⁴There is a vast literature dealing with mergers of firms selling complementary goods. See, for example, Matutes and Regibeau (1992), Economides and Salop (1992) or Bart (2008).

therefore, are more effective in deterring entry. Shopping malls would be driven out of the market by department stores because when there is competition between department stores and shopping malls, the former have higher profits.

We also compare the consumers' surplus and the total surplus in the different scenarios of retail organization. Since all the consumers are assumed to buy exactly one unit of each good, a change in prices simply transfers surplus between consumers and producers. Therefore, total surplus is maximized when consumers shop at the closest shopping center (transportation costs are minimized). This occurs when there are either two department stores or two shopping malls. Unsurprisingly, the consumers' surplus is the highest in the case of competition between two department stores. The equilibrium of the merger game (two shopping malls) is actually the worst scenario for consumers. In spite of having to support higher transportation costs, consumers are better off when there is a department store and a shopping mall than when there are two shopping malls.

Our model is pioneer in extending the spatial competition model (Hotelling, 1929; d'Aspremont, Gabszewicz and Thisse, 1979) to analyze multi-product competition between department stores and shopping malls. To the best of our knowledge, only Lal and Matutes (1989) have presented a multi-product version of the model of Hotelling (1929).⁵ They restricted the analysis to the case of competition between two department stores that sell two goods. We have greatly generalized their analysis by allowing a finite number of goods and an alternative mode of retail: the shopping mall.⁶

Other authors have analyzed multi-product price competition, but no one used the spatial competition model to do so. Moreover, most of them based the analysis on the assumption that consumers make all their purchases at the same shopping center (Bliss, 1988; Beggs, 1994; Smith and Hay, 2005; Innes, 2006). They support this "one stop shopping" assumption on the fact that shopping implies time and transportation costs. They argue that, in order to save costs, customers make all their purchases at the same

⁵There are other extensions of the spatial competition model that allow for multi-product firms, but in which consumers only buy one of the goods that are available (Laussel, 2006; Giraud-Heraud, Ham-moudi and Mokrane, 2003). Goods available in a shopping center are, in this case, substitutes instead of complements. These models correspond to completely different economic settings.

⁶In the model of Lal and Matutes (1989), there are two types of consumers: the poor and the rich. The poor do not support transportation costs, therefore, they buy each good where it is cheaper ("one stop shopping" is not assumed). The rich, on the other hand, support transportation costs and, in equilibrium, are not interested in shopping around. Their focus is to study price discrimination across the two segments.

place. In our opinion, even with the support of empirical works as the one of Rhee and Bell (2002), who have found that consumers make 94% of their weekly groceries expenditures at the same supermarket, the assumption that consumers necessarily make all their purchases at the same store is too strong. We have relaxed this hypothesis, allowing consumers to shop in more than one place. In the end, we have validated the “one stop shopping” hypothesis, but as a result.

The paper that is closest to ours is perhaps that of Beggs (1994), who studied a similar merger game in a model in which firms face a linear demand function. He restricted the analysis to the case of two goods and, as mentioned before, assumed that consumers purchase both goods at the same location (“one stop shopping” assumption). Smith and Hay (2005) have also studied price competition under alternative modes of retail organization (shopping streets, shopping malls and department stores), but they did not consider competition between the different modes.

The remainder of the article is organized as follows. In Section 2, we setup the model, introduce notation and obtain the demand and the profit functions. In Section 3, we present the possible competitive scenarios and find the equilibrium prices in each one. Section 4 is dedicated to a welfare analysis. We study the merger game in Section 5. Section 6 concludes the article with some remarks. The proofs of all propositions are collected in the Appendix.

2 The model

2.1 Basic setup

We consider a multi-product version of the model of Hotelling (1929). There is a continuum of consumers uniformly distributed across a linear city, $[0, 1]$. Each consumer buys one unit of each of the products, $i \in \{1, \dots, n\} = \mathcal{I}$, which are sold at the extremes of the city ($x = 0$ and $x = 1$). The price of good i at the left extreme (L) is denoted by p_{iL} and the price of good i at the right extreme (R) is denoted by p_{iR} .

The reservation price for each product, V_i , is assumed to be high enough for the market to

be fully covered. Thus, the demand is perfectly inelastic and the only decision of consumers is where to buy each product. Each consumer chooses among three possibilities:

(*L*) to buy all the goods at $x = 0$;

(*R*) to buy all the goods at $x = 1$;

(*LR*) to travel to both extremes and buy each good where it is cheaper.

We denote by P_L and by P_R the price that a consumer pays for all the goods at $x = 0$ and at $x = 1$, respectively ($P_L = \sum_{i=1}^n p_{iL}$ and $P_R = \sum_{i=1}^n p_{iR}$). By P_{LR} , we denote the price that a consumer pays for all the goods if she buys each good where it is cheaper ($P_{LR} = \sum_{i=1}^n \min\{p_{iL}, p_{iR}\}$).

To make their decision, consumers take into account not only the prices charged for the products, but also the transportation costs that they must support to acquire them. We assume that the transportation costs are linear in distance. Let $u_L(x)$, $u_R(x)$ and $u_{LR}(x)$ denote the utility attained by an agent located at $x \in [0, 1]$ who chooses to purchase, respectively: (*L*) all the goods at $x = 0$; (*R*) all the goods at $x = 1$; (*LR*) each good where it is cheaper. Then:

$$\begin{aligned} u_L(x) &= \sum_{i=1}^n V_i - P_L - tx, \\ u_R(x) &= \sum_{i=1}^n V_i - P_R - t(1 - x), \\ u_{LR}(x) &= \sum_{i=1}^n V_i - P_{LR} - t. \end{aligned}$$

It is fundamental to keep in mind that if a consumer travels to both extremes, she supports higher transportation costs than if she had chosen to purchase both goods at the same location. For this reason, the demand for each product at a certain location is related to the demand for any other product at any location. Products sold at the same location are complementary goods, while products sold at different locations are substitutes.

2.2 Demand and profit functions

The consumers that are most likely to purchase a good that is sold at one of the extremes are those that are located closer to that extreme. When all the goods have strictly positive demand at both locations, the consumers near the left extreme are surely buying all the goods at $x = 0$ (their choice is *L*), while those near the right extreme are surely buying all the goods at $x = 1$ (their choice is *R*).

Depending on the prices charged for each good at each location, the consumers near the middle may find it worthwhile to travel to both extremes of the city, to buy each good where it is cheaper. This occurs if some goods are sufficiently cheaper at $x = 0$ while other goods are sufficiently cheaper at $x = 1$. On the contrary, if the price differences across locations are relatively small, then all the consumers make their purchases at a single location, either at $x = 0$ or at $x = 1$.

The possible demand scenarios are illustrated in Figure 1.

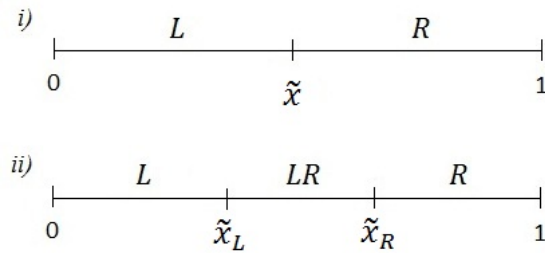


Figure 1: Possible demand scenarios.

To obtain the demand for each good at each location, it is useful to find the location of the consumer that is indifferent between each pair of choices (among L , R and LR). Accordingly, we use some additional notation.

By \tilde{x}_L , we denote the location of the consumer that is indifferent between L and LR :

$$u_L(\tilde{x}_L) = u_{LR}(\tilde{x}_L) \Leftrightarrow \tilde{x}_L = 1 - \frac{P_L - P_{LR}}{t}. \quad (1)$$

We denote by \tilde{x}_R the consumer that is indifferent between R and LR :

$$u_R(\tilde{x}_R) = u_{LR}(\tilde{x}_R) \Leftrightarrow \tilde{x}_R = \frac{P_R - P_{LR}}{t}. \quad (2)$$

Finally, we denote by \tilde{x} be the consumer that is indifferent between L and R . It is clear from the expression below that $\tilde{x} = \frac{\tilde{x}_L + \tilde{x}_R}{2}$:

$$u_L(\tilde{x}) = u_R(\tilde{x}) \Leftrightarrow \tilde{x} = \frac{1}{2} + \frac{P_R - P_L}{2t}. \quad (3)$$

There will be consumers traveling to both extremes of the city if and only if $\tilde{x}_L < \tilde{x}_R$, which is equivalent to $\sum_{i \in \mathcal{I}} |p_{iL} - p_{iR}| > t$. Otherwise, all the consumers will make their

purchases at a single place. It is easy to verify that $\sum_{i \in \mathcal{I}} |p_{iL} - p_{iR}| \leq t$ implies that $0 \leq \tilde{x} \leq 1$. Therefore, in this case, the demand for any good sold at L is \tilde{x} and the demand for any good sold at R is $1 - \tilde{x}$.

It is convenient to denote the vector of prices of all the goods at both locations by $\mathbf{p} \in \mathbb{R}_+^{2n}$ and to consider the following sets:

$$\begin{aligned} S_1 &= \{ \mathbf{p} \in \mathbb{R}_+^{2n} : \sum_{i \in \mathcal{I}} |p_{iL} - p_{iR}| \leq t \}; \\ S_2 &= \{ \mathbf{p} \in \mathbb{R}_+^{2n} : \sum_{i \in \mathcal{I}} |p_{iL} - p_{iR}| > t \}. \end{aligned}$$

If there are consumers that travel to both extremes, the demand for a good depends on whether this good is cheaper at L or at R . Denoting by \mathcal{I}_L and \mathcal{I}_R the sets of goods that are strictly cheaper at L and R , respectively, we can write the expressions for the indifferent consumers as follows:

$$\tilde{x}_L = 1 - \frac{1}{t} \sum_{i \in \mathcal{I}_R} (p_{iL} - p_{iR}) \quad (4)$$

and

$$\tilde{x}_R = \frac{1}{t} \sum_{i \in \mathcal{I}_L} (p_{iR} - p_{iL}). \quad (5)$$

The demand for a good $i \in \mathcal{I}_L$ at L is $\min \{ \tilde{x}_R, 1 \}$, while its demand at R is $\max \{ 0, 1 - \tilde{x}_R \}$. If $i \in \mathcal{I}_R$, its demand at L is $\max \{ 0, \tilde{x}_L \}$ and its demand at R is $\min \{ 1 - \tilde{x}_L, 1 \}$. In case of a tie ($p_{iL} = p_{iR}$), each consumer that travels to both extremes may either buy good i at L or at R . Any tie-breaking assumption leads to the same results. We can assume, for example, that half of the consumers buys good i at L and the other half buys it at R .

We find that the demand for good i at $x = 0$ is:

$$q_{iL} = \begin{cases} \tilde{x} & \text{if } \mathbf{p} \in S_1 \\ \min \{ \tilde{x}_R, 1 \} & \text{if } \mathbf{p} \in S_2 \wedge p_{iL} < p_{iR} \\ \frac{1}{2} (\min \{ \tilde{x}_R, 1 \} + \max \{ 0, \tilde{x}_L \}) & \text{if } \mathbf{p} \in S_2 \wedge p_{iL} = p_{iR} \\ \max \{ 0, \tilde{x}_L \} & \text{if } \mathbf{p} \in S_2 \wedge p_{iL} > p_{iR} \end{cases},$$

while the demand for the same good at $x = 1$ is $q_{iR} = 1 - q_{iL}$.

Without loss of generality, the marginal cost of producing one unit of each of the goods

is assumed to be zero. Under this assumption, the profits coincide with the sales revenues. This simplification does not affect any of the results in the paper.⁷

The profit that results from selling good i at $x = 0$ is:

$$\Pi_{iL} = \begin{cases} p_{iL} \left(\frac{1}{2} + \frac{P_R - P_L}{2t} \right) & \text{if } \mathbf{p} \in S_1 \\ p_{iL} \min \left\{ \frac{P_R - P_{LR}}{t}, 1 \right\} & \text{if } \mathbf{p} \in S_2 \wedge p_{iL} < p_{iR} \\ \frac{p_{iL}}{2} \left(\min \left\{ \frac{P_R - P_{LR}}{t}, 1 \right\} + \max \left\{ 0, 1 - \frac{P_L - P_{LR}}{t} \right\} \right) & \text{if } \mathbf{p} \in S_2 \wedge p_{iL} = p_{iR} \\ p_{iL} \max \left\{ 0, 1 - \frac{P_L - P_{LR}}{t} \right\} & \text{if } \mathbf{p} \in S_2 \wedge p_{iL} > p_{iR} \end{cases}.$$

By symmetry, the profit that results from selling good i at the right extreme of the city is:

$$\Pi_{iR} = \begin{cases} p_{iR} \left(\frac{1}{2} + \frac{P_L - P_R}{2t} \right) & \text{if } \mathbf{p} \in S_1 \\ p_{iR} \min \left\{ \frac{P_L - P_{LR}}{t}, 1 \right\} & \text{if } \mathbf{p} \in S_2 \wedge p_{iR} < p_{iL} \\ \frac{p_{iR}}{2} \left(\min \left\{ \frac{P_L - P_{LR}}{t}, 1 \right\} + \max \left\{ 0, 1 - \frac{P_R - P_{LR}}{t} \right\} \right) & \text{if } \mathbf{p} \in S_2 \wedge p_{iR} = p_{iL} \\ p_{iR} \max \left\{ 0, 1 - \frac{P_R - P_{LR}}{t} \right\} & \text{if } \mathbf{p} \in S_2 \wedge p_{iR} > p_{iL} \end{cases}.$$

We have obtained the demand and the profit associated with each good at each location, as a function of prices. The actual (equilibrium) prices are determined in the next section.

3 Competitive scenarios

Now, we focus on the supply side, considering the following modes of retail:

DM - a department store at $x = 0$ and a shopping mall at $x = 1$;

DD - two department stores, one at $x = 0$ and another at $x = 1$;

MM - two shopping malls, one at $x = 0$ and another at $x = 1$.

We designate by department store a multi-product firm that sells the n goods at the same location. For example, a department store at $x = 0$ sells goods $\{iL\}_{i \in \mathcal{I}}$, seeking to maximize its profit, $\Pi_L = \sum_{i=1}^n \Pi_{iL}$.

By shopping mall, we mean a group of single-product firms that sell each of the n goods at the same location. For example, a shopping mall at $x = 1$ is composed by n firms,

⁷To obtain the equilibrium prices for the case in which the marginal costs are different from zero (being equal across locations), simply add the marginal costs to the equilibrium prices that we obtain.

with each firm selling one good, iR , with the objective of maximizing its individual profit, Π_{iR} . We exclude the possibility of concerted behavior between shops at a mall. Each shop chooses how much to charge for the product it sells, taking the remaining prices as given.

3.1 Competition between a department store and a shopping mall

We start by considering the case in which there is a department store located at $x = 0$ and a shopping mall located at $x = 1$. The department store chooses the prices of the n goods with the objective of maximizing its total profit ($\sum_{i=1}^n \Pi_{iL}$), while each of the shops at the mall seeks to maximize its individual profit (Π_{iR}).

The profit of the department store is given by:⁸

$$\Pi_L = \begin{cases} P_L \left(\frac{1}{2} + \frac{P_R - P_L}{2t} \right), & \mathbf{p} \in S_1 \\ \tilde{x}_L \sum_{i \in \mathcal{I}_R} p_{iL} + \tilde{x}_R \sum_{i \in \mathcal{I}_L} p_{iL}, & \mathbf{p} \in S_2 \wedge \tilde{x}_L \in [0, 1] \wedge \tilde{x}_R \in [0, 1] \wedge \mathcal{I}_L \cup \mathcal{I}_R = \mathcal{I} \end{cases} .$$

The following result is instrumental. It states that if the department store sets profit-maximizing prices, there is “one stop shopping”.

Lemma 1. *Independently of the prices set at the right extreme, $\{p_{iR}\}_{i \in \mathcal{I}} \in \mathbb{R}_+^n$, the prices that maximize the profit of the department store, Π_L , are such that $\mathbf{p} \in S_1$.*

Proof. See Appendix. □

Proposition 1. *In the case of competition between a department store and a shopping mall:*

(1) *It is cheaper to buy the n goods at the department store than at the shopping mall:*

$$\begin{cases} P_L = \sum_{i=1}^n p_{iL} = \frac{2n+1}{n+2}t \\ P_R = \sum_{i=1}^n p_{iR} = \frac{3n}{n+2}t \end{cases} ,$$

⁸The branches where $\tilde{x}_L \notin [0, 1]$ and $\tilde{x}_R \notin [0, 1]$ are omitted because they are irrelevant. The department store would never choose such prices. For the same reason, we also omit the case in which $\mathbf{p} \in S_2$ and $\exists i \in \mathcal{I} : p_{iL} = p_{iR} > 0$. The department store would gain by reducing p_{iL} infinitesimally.

with $p_{iR} = \frac{3}{n+2}t$, $\forall i \in \mathcal{I}$, and $\sum_{i=1}^n |p_{iL} - p_{iR}| \leq \frac{n+6\sqrt{2}-7}{n+2}t < t$.

(2) The demand is greater at the department store and no consumer shops at both extremes of the city:

$$\begin{cases} q_{iL} = \frac{2n+1}{2n+4} \\ q_{iR} = \frac{3}{2n+4} \end{cases}.$$

(3) The department store earns more profits than the shops at the mall taken together:

$$\begin{cases} \Pi_L = \sum_{i=1}^n \Pi_{iL} = \frac{(2n+1)^2}{2(n+2)^2}t \\ \Pi_R = \sum_{i=1}^n \Pi_{iR} = \frac{9n}{2(n+2)^2}t \end{cases}.$$

Proof. See Appendix. □

In equilibrium, the prices set by the department store and the shops at the mall are such that no consumer is willing to travel to both extremes of the city. As all customers buy the entire bundle of goods at the same place, the department store does not care about how much to charge for each good. What matters for the department store is the price of the bundle.⁹

We have also found that it is cheaper to buy the n products at the department store than at the shopping mall. This happens because the department store has an additional incentive to set low prices. By decreasing the price of one good, for example, the price of books, the department store increases the demand for all goods sold there (books, groceries, etc.). At the shopping mall, the bookshop, when choosing the price to set for books, only takes into account the effect on its own demand, ignoring the effect of the price of books on the demand for groceries and for the remaining goods.

As a result of setting lower prices, the department store captures more than half of the market. It does not capture the whole market because the customers that are closer to the shopping mall must weight the price advantage of the department store against the proximity advantage of the shopping mall. In equilibrium, the shopping mall retains the consumers that are sufficiently close.

Comparing the joint profit at each extreme of the city, we find that the department

⁹The indeterminacy of prices at the department store does not extend to the demand.

store earns more than the rival firms taken together. Aware of this, the shops at the mall could believe that a merger would benefit them, that is, increase their joint profit. In the next subsection, we study the effects of such a merger.

Observe also that as the number of products increases to infinity, the price of the bundle increases to $2t$ at the department store and to $3t$ at the shopping mall. Moreover, the higher is n , the higher is the difference between the price of the bundle of goods in the two extremes. As a result, the department store captures an increasing share of the market (in the limit, it actually captures the whole market). Profits at the department store increase to $2t$ while the profits at the shopping mall initially increase (are maximal for $n = 2$) but then decrease to zero.

When the number of products tends to infinity, the department store becomes monopolistic in the market, since all consumers purchase the bundle there. However, the price she charges for the bundle remains bounded by $2t$, because additional price increases would lead to a loss of consumers to the shopping mall. Then, although not buying from the shopping mall, consumers benefit from its existence.

3.2 Competition between two department stores

Now, we consider the case in which there are two department stores, one at each extreme of the city. Each department store chooses the price to charge for each of the n products, with the objective of maximizing its profit, taking as given the prices set by the other department store.¹⁰

Proposition 2. *In the case of competition between two department stores:*

(1) *The price of the bundle is equal to the transportation cost parameter:*

$$P_L = P_R = t, \text{ with } \sum_{i=1}^n |p_{iL} - p_{iR}| \leq t.$$

¹⁰This is an extension of the case analyzed by Lal and Matutes (1989), where both department stores sell only two products ($n = 2$).

(2) Consumers make all their purchases at the closest department store, thus:

$$q_{iL} = q_{iR} = \frac{1}{2}.$$

(3) The resulting profits are also independent of the number of goods:

$$\Pi_L = \Pi_R = \frac{t}{2}.$$

Proof. See Appendix. □

Again, no consumer is willing to travel to both extremes of the city. They all buy the n goods at the department store that is closer.

As expected, the department stores charge the same price for the bundle of n goods (there is, once more, some indeterminacy regarding the split of the bill between the goods). What is a bit surprising is that the margin (difference between price and marginal cost) obtained with n goods is the same as that obtained in the single-product model. The margin is not greater with n products than in the case of a single product because the reservation utility of the customer is not relevant for the pricing decisions of the firms (as long as it is high enough, as typically assumed). Despite the fact that customers prefer n goods instead of one, the margin remains constant and equal to the transportation cost parameter.¹¹

As a result, the two department stores obtain exactly the same profit as in the standard Hotelling model, in which a single good is sold. Comparing these profits with those obtained in the previous scenario, we find that transforming a shopping mall into a department store is not profitable. The joint profit of the n shops at the mall that compete with a department store is greater than the profit of a department store that competes with another department store.

Hence, in spite of the fact that the department store obtains a higher profit than the shops at the mall taken together, it is not profitable for the shops at the mall to merge into a department store. To put it in another way, to compete with a department store it

¹¹The same would occur in the case of Bertrand competition with homogeneous products. Independently of the number of products that firms sell, their equilibrium margin is always null.

is better to construct a shopping mall than to construct another department store.

This result makes us wonder whether it is not better for the department store that sells the n goods to separate into a shopping mall with n single-product shops. We investigate this possibility in the next subsection.

3.3 Competition between two shopping malls

In the case of competition between two shopping malls (one at each extreme of the city), there are $2n$ independent stores that maximize their individual profits.

The stores selling the same good in different locations are direct competitors. However, the demand of a store also depends on the price of the other goods sold in the mall where it is located. A store that sells good i benefits from: (i) a low price for the good j sold at the same location (since this attracts customers to its location); (ii) a high price for the good j sold at the other location (since this repels customers from the other location). This interdependence across goods occurs because when deciding where to buy each good, a customer takes into account not only the price but also the transportation costs that she has to support.

Lemma 2. *When there are two shopping malls in the city, no consumer shops at both extremes of the city (in equilibrium).*

Proof. See Appendix. □

Combining Lemma 1 and Lemma 2, we conclude that no consumer finds it worthwhile to travel to both extremes of the city.

Corollary 1. *In equilibrium, regardless of the mode of retail, there is “one stop shopping”.*

Proposition 3. *In the case of competition between two shopping malls:*

(1) *The price of each good is equal to the transportation cost parameter:*

$$p_{iL} = p_{iR} = t, \quad \forall i \in \mathcal{I}.$$

(2) *Consumers make all their purchases at the closest shopping mall:*

$$q_{iL} = q_{iR} = \frac{1}{2}, \quad \forall i \in \mathcal{I}.$$

(3) *The profit of each firm is also independent of the number of goods:*

$$\Pi_{iL} = \Pi_{iR} = \frac{t}{2}, \quad \forall i \in \mathcal{I}.$$

Proof. See Appendix. □

The joint profit of the n shops located at each shopping mall is equal to $nt/2$, which is greater than the profits obtained in any of the alternative scenarios that we have considered. Hence, it is profitable for the department store to separate into several shops.

4 Welfare analysis

4.1 Social welfare

In our model, demand is perfectly inelastic, since each consumer buys one unit of each good that is available in the market. Moreover, consumers' reservation prices ($V_i, i \in \mathcal{I}$) are assumed to be high enough to ensure full coverage of the market. In this context, a change in prices only leads to a transfer of surplus between consumers and firms. As a result, the total surplus only depends on the transportation costs incurred by the consumers. The maximization of total surplus is equivalent to the minimization of total transportation costs.

It is well known that the total transportation costs are minimized when each consumer shops at the closest store. This occurs both in the case of competition between two department stores and in the case of competition between two shopping malls. When there is a department store competing with a shopping mall, the indifferent consumer is no longer located at the middle of the city. There are more consumers shopping at the department store than at the shopping mall. The existence of different modes of retail diminishes the total surplus.

4.2 Consumers' surplus

From the point of view of consumers, competition between department stores is the most favorable scenario. Prices are lower than in the other scenarios, and transportation costs are minimized. It is not so straightforward to compare the case of competition between two shopping malls (lower transportation costs) with the case of competition between a shopping mall and a department store (lower prices).

The total consumers' surplus is given by:¹²

$$CS = \tilde{x}(V - P_L) + (1 - \tilde{x})(V - P_R) - \int_0^{\tilde{x}} tx \, dx - \int_{\tilde{x}}^1 t(1 - x) \, dx,$$

where $V = \sum_{i=1}^n V_i$.

Let CS_{DD} , CS_{MM} and CS_{DM} denote the consumers' surplus in the three different scenarios: (DD) competition between two department stores; (MM) competition between two shopping malls; and (DM) competition between a department store and a shopping mall.

Recall that when the mode of retail is the same in both extremes of the city, the price of the basket of goods is equal at both extremes. As a result, the indifferent consumer is located at the middle of the city and the total transportation cost is minimized. When there are two department stores, the price of the basket is t . When there are two shopping malls, the basket costs nt . Consequently:

$$CS_{DD} = \frac{1}{2}(V - t) + \frac{1}{2}(V - t) - t \int_0^{\frac{1}{2}} x \, dx - t \int_{\frac{1}{2}}^1 (1 - x) \, dx = V - \frac{5}{4}t$$

and

$$CS_{MM} = \frac{1}{2}(V - nt) + \frac{1}{2}(V - nt) - t \int_0^{\frac{1}{2}} x \, dx - t \int_{\frac{1}{2}}^1 (1 - x) \, dx = V - \left(n + \frac{1}{4}\right)t.$$

¹²Recall that the "one stop shopping" condition holds. Then, consumers located at $[0, \tilde{x}[$ purchase all goods at $x = 0$, while consumers located at $]\tilde{x}, 1]$ make all purchases at $x = 1$.

When a department store competes with a shopping mall, the consumers' surplus is:

$$CS_{DM} = V - \frac{10n^2 + 28n + 7}{4(n+2)^2}t.$$

Our conclusions can be summarized by the following result.

Proposition 4. *Comparing the consumer' surplus in the three competitive scenarios, for $n > 1$, we obtain:*

$$CS_{DD} > CS_{DM} > CS_{MM}.$$

It is somewhat surprising that the lower the number of independent stores in the market, the higher the consumers' surplus. This result contradicts the typical intuition, according to which as the number of firms in the market increases, competition becomes stronger, leading to lower prices. This is not the case, since the price for the basket of the two goods is cheaper when there are only two department stores.

5 Merger game

Until now, we have assumed that the mode of retail was exogenous, that is, stores in each extreme of the city took their mode of organization (in a department store or in a shopping mall) as a given. In this section, we make the mode of retail endogenous, making it a decision of the shops. We analyze whether the shops at a mall consider their merger profitable, assuming that they decide to merge if the post-merger profit is higher than the sum of the individual profits before the merger.¹³

In other words, we determine the equilibrium of the following merger game:

1st stage - The n shops at each extreme simultaneously decide whether to merge or not;

2nd stage - Given the modes of retail at both extremes, the stores simultaneously choose prices.

¹³We assume that, in the case of a merger, the shops receive equal shares of the total profit.

We only allow for two outcomes in the first stage: *full merger* or *no merger*. We do not consider the possibility of a merger between k stores located at one extreme while the other $n - k$ stores remain separated. This would be an interesting case to study, but it is out of our scope. Moreover, we restrict the analysis to mergers between stores at the same extreme of the city. Otherwise, a store selling the product $i \in \mathcal{I}$ at $x = 0$ could merge with a store selling the same product at $x = 1$ to form a monopoly in this market.

We find the equilibrium of the game by backward-induction. The solutions of the second stage were already determined in section 3. The results are summarized in Table 1.

	Department Store	Shopping Mall
Department Store	$\left(\frac{t}{2}, \frac{t}{2}\right)$	$\left(\frac{(2n+1)^2}{2(n+2)^2}t, \frac{9n}{2(n+2)^2}t\right)$
Shopping Mall	$\left(\frac{9n}{2(n+2)^2}t, \frac{(2n+1)^2}{2(n+2)^2}t\right)$	$\left(\frac{nt}{2}, \frac{nt}{2}\right)$

Table 1: Joint profits at each extreme of the city.

The next proposition follows immediately from Table 1.

Proposition 5. *The unique equilibrium of the merger game is such that the stores at both extremes do not merge.*

In equilibrium, we should expect to have a shopping mall at both extremes of the city. Not to merge is actually a dominant strategy in this game. With our formulation, stores never have an incentive to merge. We should not observe (as we do in reality) the existence of department stores. The reality is, for sure, much more complex than our model.

6 Conclusions

We have developed a multi-product version of the model of Hotelling (1929) to study competition between shopping centers that can be organized as department stores or as shopping malls. In particular, we analyzed how the mode of retail affects prices, market shares and profits.

A distinctive feature of our work, with respect to the existing literature, is that we do not restrict consumers to make all their purchases in a single place. However, we found that, in equilibrium, no consumer finds it worthwhile to visit more than one shopping center. Since we have obtained the “one-stop shopping” condition as an equilibrium outcome, our work provides a theoretical justification for this commonly made assumption.

Comparing the competitive behaviour of a department store with that of a shopping mall, we found (as in previous works) that the department store competes more aggressively (charges a lower price for the bundle of products). This occurs because a department store, when choosing prices, takes into account the fact that a price drop in one good increases the demand for all its goods. In contrast, a shop at a mall only takes into account its individual demand when choosing the price to charge for its good.

When a department store competes with a shopping mall, the bundle of goods is cheaper at the department store. Nevertheless, the demand-effect more than compensates the price-effect and the department store obtains higher profits than the shops at the mall taken together. As the number of goods increases, consumers worry more and more about price differences across shopping centers (and less and less about transportation costs). In the limit, the department store actually captures the whole market.

In spite of having a lower profit, the shops at mall have no incentives to merge into a department store. It is the department store that has incentives to separate itself into a shopping mall. The equilibrium of a merger game, in which the shops at each extreme decide whether to organize themselves as a shopping mall or as a department store, is competition between two shopping malls. In other words, if the mode of retail organization is made endogenous, only shopping malls are expected to appear.

Having two shopping malls or two department stores is equally optimal in terms of total surplus. However, consumers are better off in the case of competition between two department stores (since they pay less for the bundle). Therefore, the equilibrium of the merger game (two shopping malls) is the competitive scenario that consumers less desire.

7 Appendix

Proof of Lemma 1.

Assume, by way of contradiction, that the prices that maximize the profits of a department store at L (given the prices at R) are such that $\mathbf{p} \in S_2$.

(i) We start by considering the possibility of an interior solution, assuming that $0 < \tilde{x}_L < \tilde{x} < \tilde{x}_R < 1$ and that $p_{iL} \neq p_{iR}$, $\forall i \in \mathcal{I}$.

In this case, the profit function is continuous with respect to p_{iL} , $\forall i \in \mathcal{I}$, in a neighborhood of \mathbf{p} . This implies that the first-order conditions must be satisfied.

In the interior of any branch in S_2 , the profit of the department store is given by:

$$\Pi_L = \tilde{x}_L \sum_{j \in \mathcal{I}_R} p_{jL} + \tilde{x}_R \sum_{j \in \mathcal{I}_L} p_{jL} = \tilde{x}_L \sum_{j \in \mathcal{I}} p_{jL} + (\tilde{x}_R - \tilde{x}_L) \sum_{j \in \mathcal{I}_L} p_{jL}.$$

Using the fact that $\tilde{x} - \tilde{x}_L = \tilde{x}_R - \tilde{x}$, we can write:

$$\Pi_L = \tilde{x} \sum_{j \in \mathcal{I}} p_{jL} + (\tilde{x} - \tilde{x}_L) \left(\sum_{j \in \mathcal{I}_L} p_{jL} - \sum_{j \in \mathcal{I}_R} p_{jL} \right).$$

The first-order conditions, for $i \in \mathcal{I}_R$, are:

$$\frac{\partial \Pi_L}{\partial p_{iL}} = 0 \Leftrightarrow \tilde{x}_L = \frac{1}{t} \sum_{j \in \mathcal{I}_R} p_{jL} \Leftrightarrow 2 \sum_{j \in \mathcal{I}_R} p_{jL} = t + \sum_{j \in \mathcal{I}_R} p_{jR},$$

while, for $i \in \mathcal{I}_L$, the first-order conditions are:

$$\frac{\partial \Pi_L}{\partial p_{iL}} = 0 \Leftrightarrow \tilde{x}_R = \frac{1}{t} \sum_{j \in \mathcal{I}_L} p_{jL} \Leftrightarrow 2 \sum_{j \in \mathcal{I}_L} p_{jL} = \sum_{j \in \mathcal{I}_L} p_{jR}.$$

Using the two last equalities, we can rewrite the expression of Π_L as follows:

$$\Pi_L = \tilde{x} P_L - \frac{t}{2} (\tilde{x} - \tilde{x}_L).$$

If the department store had chosen prices such that $\mathbf{p} \in S_1$ with the same P_L , its profit

would be $\tilde{x}P_L$, which is greater. For example, the department store could set $p_{iL} = ap_{iR}$, $\forall i \in \mathcal{I}$, where $a = \frac{P_L}{P_R}$. To check that these prices would belong to S_1 , observe that, from the FOC, we have $2P_L = t + P_R$, which implies that $a - 1 = \frac{t}{2P_R}$. Thus, $\sum_{i \in \mathcal{I}} |p_{iL} - p_{iR}| = \frac{t}{2} \leq t$. Contradiction.

(ii) We now consider the possibility of a boundary solution.

If $\exists i \in \mathcal{I} : p_{iL} = p_{iR} > 0$, the department store would gain by reducing p_{iL} infinitesimally. If $\exists i \in \mathcal{I} : p_{iL} = p_{iR} = 0$, then a small increase of p_{iL} and a decrease in the same amount of some p_{jL} , with $j \in \mathcal{I}_R$, would keep the profit constant (observe that P_L and P_{LR} remain constant). Therefore, we can assume, without loss of generality, that $p_{iL} \neq p_{iR}$, $\forall i \in \mathcal{I}$.

We must have, therefore, $\tilde{x}_L = 0$ or $\tilde{x}_R = 1$.

Suppose that $\tilde{x}_L = 0$. The fact that the maximum occurs at this boundary implies that the derivative of the profit function with respect to p_{iL} , with $i \in \mathcal{I}_R$, is positive. But revenues from these goods are already null, thus increasing prices cannot compensate. Contradiction.

We are left with the case in which $\tilde{x}_R = 1$. The derivative of the profit function with respect to p_{iL} , with $i \in \mathcal{I}_L$, must be negative. Decreasing prices would compensate if it weren't for the fact that demand is bounded at 1. If the demand continued to increase, the profit would be maximized at the point in which the FOC are satisfied. Even in this case, as we verified in (i), it would be better to choose prices such that $\mathbf{p} \in S_1$ with the same P_L . Therefore, the maximum is surely not at S_2 with $\tilde{x}_R = 1$. \square

Proof of Proposition 1.

As there is a department store at $x = 0$, by Lemma 1 we know that there is no equilibrium with prices in S_2 . Therefore, we must seek equilibrium prices in S_1 , that is, prices such that $\sum_{i=1}^n |p_{iL} - p_{iR}| \leq t$.

In S_1 , the first-order conditions for the profit-maximization of the department store imply that:

$$P_L = \frac{P_R}{2} + \frac{t}{2}. \quad (6)$$

Since, by Lemma 1, the maximum is attained in S_1 , this is a global maximum.

The first-order condition for the profit-maximization of the shop that sells good $i \in \mathcal{I}$ at the mall is:

$$2p_{iR} = t + P_L - \sum_{j \neq i} p_{jR}. \quad (7)$$

Adding up the first-order conditions of the n shops at the mall, we obtain:

$$2P_R = nt + nP_L - (n-1)P_R \Leftrightarrow P_R = \frac{n}{n+1}(t + P_L). \quad (8)$$

Combining (6) with the first-order conditions of the shops at the mall, (7), we obtain the (candidate) equilibrium:

$$P_L^* = \frac{2n+1}{n+2}t, \quad P_R^* = \frac{3n}{n+2}t \quad \text{and} \quad p_{iR}^* = \frac{3}{n+2}t. \quad (9)$$

To finish the proof, we must verify that $p_{iR}^* = \frac{3}{n+2}t$ maximizes the profit of the shop at the mall that sells good iR . So far, we only know that it is a maximum in the branch that corresponds to S_1 .

To study the behavior of Π_{iR} as a function of p_{iR} , it is convenient to define the following partition of the domain of p_{iR} :

$$\begin{aligned} D_1 &= [0, -t + p_{iL} + s_R]; \\ D_2 &=]-t + p_{iL} + s_R, -t + p_{iL} + s_L + s_R[; \\ D_3 &= [-t + p_{iL} + s_L + s_R, t + p_{iL} - s_L - s_R]; \\ D_4 &=]t + p_{iL} - s_L - s_R, t + p_{iL} - s_L[; \\ D_5 &= [t + p_{iL} - s_L, +\infty[, \end{aligned}$$

where $s_L = \sum_{j \in \mathcal{I}_L \setminus \{i\}} (p_{jR} - p_{jL})$ and $s_R = \sum_{j \in \mathcal{I}_R \setminus \{i\}} (p_{jL} - p_{jR})$.

The demand for good iR , as a function of p_{iR} , satisfies:

$$q_{iR} = \begin{cases} 1, & p_{iR} \in D_1 \\ \frac{1}{t} \sum_{j \in \mathcal{I}_R} (p_{jL} - p_{jR}), & p_{iR} \in D_2 \\ \frac{1}{2} + \frac{P_L}{2t} - \frac{P_R}{2t}, & p_{iR} \in D_3 \\ 1 - \frac{1}{t} \sum_{j \in \mathcal{I}_L} (p_{jR} - p_{jL}), & p_{iR} \in D_4 \\ 0, & p_{iR} \in D_5 \end{cases}. \quad (10)$$

The demand is linear in each branch and globally continuous. Its slope is initially zero (in D_1), then $-\frac{1}{t}$ (in D_2), then $-\frac{1}{2t}$ (in D_3), then $-\frac{1}{t}$ again (in D_4) and, finally, zero again (in D_5).

The profit function is concave in each branch and globally continuous. It starts at zero (for $p_{iR} = 0$) and ends at zero (for $p_{iR} \in D_5$). With $P_L^* = \frac{2n+1}{n+2}t$ and $p_{jR}^* = \frac{3}{n+2}t$, $\forall j \neq i$, the profit function has a local maximum in D_3 , attained at $p_{iR}^* = \frac{3}{n+2}t$. Further price increases, to D_4 , are not profitable, since the slope of the demand is higher ($-\frac{1}{t}$ in D_4 versus $-\frac{1}{2t}$ in D_3). Therefore, the maximum is either $p_{iR}^* = \frac{3}{n+2}t$ or attained in D_2 .

In D_2 , the demand for good iR is $q_{iR} = \frac{1}{t}(s_R + p_{iL} - p_{iR})$ and the corresponding profit is $\Pi_{iR} = \frac{1}{t}(s_R p_{iR} + p_{iL} p_{iR} - p_{iR}^2)$. The first-order condition for the profit-maximization implies that: $p_{iR}^{**} = \frac{s_R}{2} + \frac{p_{iL}}{2}$. If $p_{iR}^{**} \in D_2$, the profit is:

$$\Pi_{iR}(p_{iR}^{**}) = \frac{(s_R + p_{iL})^2}{4t}.$$

As a result, $p_{iR}^* = \frac{3}{n+2}t$ is a global maximizer, if one of the two following conditions is satisfied:

i) $\Pi_{iR}(p_{iR}^{**}) < \Pi_{iR}(p_{iR}^*)$, which is true if:

$$\frac{(s_R + p_{iL})^2}{4t} < \frac{9}{2(n+2)^2}t \Leftrightarrow s_R + p_{iL} \leq \frac{\sqrt{18}}{n+2}t;$$

ii) the alternative maximizer, $p_{iR}^{**} = \frac{s_R}{2} + \frac{p_{iL}}{2}$, is outside the domain, D_2 . This occurs if:

$$-t + p_{iL} + s_R \leq \frac{s_R}{2} + \frac{p_{iL}}{2} \leq -t + p_{iL} + s_L + s_R. \quad (11)$$

In (the candidate) equilibrium, p_{iR}^* , we have $P_R - P_L = \frac{n-1}{n+2}t$. As a result,

$$\sum_{j \in I \setminus \{i\}} (p_{jR} - p_{jL}) + \frac{3}{n+2}t - p_{iL} = \frac{n-1}{n+2}t \Leftrightarrow s_L = \frac{n-4}{n+2}t + s_R + p_{iL}. \quad (12)$$

As the value of s_L does not depend on p_{iR} , it has the same value in D_2 and in D_3 . Then, substituting (12) in (11), we find that p_{iR}^{**} is outside D_2 when:

$$\frac{s_R}{2} + \frac{p_{iL}}{2} \geq -t + 2s_R + 2p_{iL} + \frac{n-4}{n+2}t \Leftrightarrow s_R + p_{iL} \leq \frac{4}{n+2}t.$$

Then, if $s_R + p_{iL} \geq \frac{\sqrt{18}}{n+2}t$, $p_{iR}^{**} = \frac{s_R}{2} + \frac{p_{iL}}{2}$ is in D_2 and, therefore, upsets our candidate equilibrium, p_{iR}^* . We conclude that p_{iR}^* is an equilibrium if and only if:

$$\forall i \in \mathcal{I}, s_R + p_{iL} \leq \frac{\sqrt{18}}{n+2}t.$$

An equivalent, but more elegant, condition is obtained by using (12):

$$\sum_{i \in \mathcal{I}} \left| p_{iL} - \frac{3}{n+2}t \right| \leq \frac{n+6\sqrt{2}-7}{n+2}t < t.$$

The equilibrium demand and profits follow immediately. □

Proof of Proposition 2.

By Lemma 1, the best response prices are such that $\sum_{i=1}^n |p_{iL} - p_{iR}| \leq t$.

The first-order conditions for the profit-maximization problems of the department stores imply that:

$$P_L = \frac{P_R}{2} + \frac{t}{2} \quad \text{and} \quad P_R = \frac{P_L}{2} + \frac{t}{2},$$

yielding:

$$P_L = P_R = t.$$

The equilibrium demand and profits follow immediately. □

Proof of Lemma 2.

By way of contradiction, suppose that the vector of prices that maximize the profits of the shops is such that $\mathbf{p} \in S_2$. More precisely, that $0 < \tilde{x}_L < \tilde{x} < \tilde{x}_R < 1$.

(i) There cannot be any $i \in \mathcal{I}$ for which $p_{iL} = p_{iR} > 0$. If that was the case, the shop selling good iL could infinitesimally reduce its price and conquer all consumers at $x \in [\tilde{x}_L, \tilde{x}_R]$. The shop selling good iR would have the same incentive to decrease its price.

It cannot also be the case that $p_{iL} = p_{iR} = 0$ for some $i \in \mathcal{I}$. In such a situation, both shops would obtain a null profit. However, the shop selling good i at $x = 0$, could choose $p_{iL} > 0$ and profit $\Pi_{iL} = p_{iL}\tilde{x}_L > 0$. The same argument applies to the shop selling good iR .

(ii) Since $p_{iL} \neq p_{iR}$, $\forall i \in \mathcal{I}$, we have $\mathcal{I}_L \cup \mathcal{I}_R = \mathcal{I}$. Thus, if the cardinality of \mathcal{I}_L is k , the cardinality of \mathcal{I}_R is $n - k$.

The profit function of the shop selling the good iL is:

$$\Pi_{iL} = \begin{cases} p_{iL} \tilde{x}_R, & i \in \mathcal{I}_L \\ p_{iL} \tilde{x}_L, & i \in \mathcal{I}_R \end{cases},$$

while the profit function of the shop selling the good iR is:

$$\Pi_{iR} = \begin{cases} p_{iR} (1 - \tilde{x}_R), & i \in \mathcal{I}_L \\ p_{iR} (1 - \tilde{x}_L), & i \in \mathcal{I}_R \end{cases}.$$

If $i \in \mathcal{I}_L$, the first-order conditions are:

$$\begin{cases} \frac{\partial \Pi_{iL}}{\partial p_{iL}} = 0 \\ \frac{\partial \Pi_{iR}}{\partial p_{iR}} = 0 \end{cases} \Leftrightarrow \begin{cases} p_{iL} = P_R - P_{LR} \\ p_{iR} = t - P_R + P_{LR} \end{cases} \Rightarrow p_{iR} = t - p_{iL}.$$

From the expressions above, we conclude that $p_{iL} = p_{jL}$, $\forall i, j \in \mathcal{I}_L$. Moreover,

$$p_{iL} = \sum_{j \in \mathcal{I}_L} (p_{jR} - p_{jL}) = k(t - 2p_{iL}) \Leftrightarrow p_{iL} = \frac{k}{2k+1}t$$

and

$$p_{iR} = \frac{k+1}{2k+1}t.$$

Analogously, if $i \in \mathcal{I}_R$, we have that:

$$p_{iL} = \frac{n-k+1}{2n-2k+1}t \quad \text{and} \quad p_{iR} = \frac{n-k}{2n-2k+1}t.$$

The expressions for the marginal consumers, \tilde{x}_L and \tilde{x}_R , follow immediately:

$$\tilde{x}_L = \frac{n-k+1}{2n-2k+1} \quad \text{and} \quad \tilde{x}_R = \frac{k}{2k+1}.$$

It is straightforward to see that $\tilde{x}_L > \tilde{x}_R$. Contradiction.

□

Proof of Proposition 3.

By Lemma 2, there is no equilibrium with prices in S_2 . Therefore, we must seek prices satisfying the condition $\sum_{i=1}^n |p_{iL} - p_{iR}| \leq t$.

As obtained in (8), the first-order conditions of the n shops at the malls imply that:

$$P_L = \frac{n}{n+1}(t + P_R) \quad \text{and} \quad P_R = \frac{n}{n+1}(t + P_L).$$

Therefore:

$$P_L = P_R = nt.$$

Using (7), we obtain the individual prices:

$$p_{iL} = p_{iR} = t, \forall i \in \mathcal{I}.$$

To complete the proof, we must verify that these local maxima are global maxima. We need to check if each shop chooses the price t , when the remainders charge t for their products. Without loss of generality, we consider the shop that sells good iR .

Substituting $p_{jL} = p_{jR} = t, \forall j \neq i$ and $p_{iL} = t$ in the demand for good iR , given in (10), we obtain:

$$q_{iR} = \begin{cases} 1 - \frac{p_{iR}}{2t}, & p_{iR} \in [0, 2t] \\ 0, & p_{iR} \in]2t, +\infty[\end{cases}.$$

The profit function is:

$$\Pi_{iR} = \begin{cases} p_{iR} \left(1 - \frac{p_{iR}}{2t}\right), & p_{iR} \in [0, 2t] \\ 0, & p_{iR} \in]2t, +\infty[\end{cases},$$

which is globally concave and continuous. Therefore, the local maximum is also the global maximum.

The equilibrium demand and profits follow immediately. □

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