The Second Malta Conference in Graph Theory and Combinatorics 2017

2MCGTC-2017

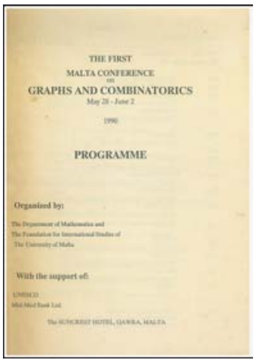
in honour of the 75^{th} ***birthday of***
Professor Stanley Fiorini

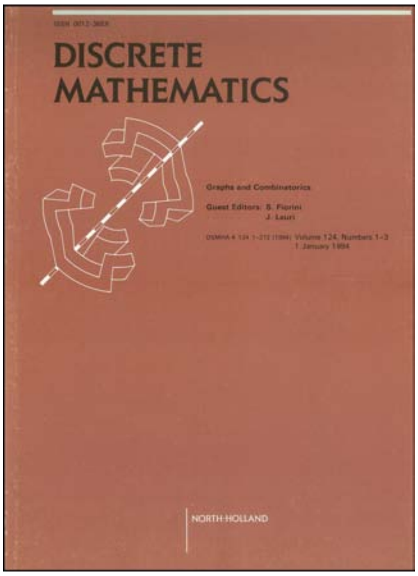
26–30 June 2017

Welcome Address

Merħba! We are honoured that you chose to join us for *The Second Malta Conference in Graph Theory and Combinatorics.* This conference is commemorating the 75^{th} birthday of **Professor Stanley Fiorini**, who introduced graph theory and combinatorics at the University of Malta.

Many of you may be asking when the previous Malta Conference was held? *The First Malta Conference on Graphs and Combinatorics* was held during the period 28 May – 2 June, 1990, at the Suncrest Hotel, also in Qawra, St Paul's Bay. It differed from a number of similar conferences held in the central Mediterranean region at that time in that it consisted of three types of lectures. László Lovász and Carsten Thomassen delivered two instructional courses of five one-hour lectures each; the former was *A survey of independent sets in graphs* and the latter was on *Embeddings of graphs.* There were also four invited speakers, namely L.W. Beineke, N.L. Biggs, R. Graham and D.J.A. Welsh, each of whom gave a one-hour lecture. The third type of talks were the 20-minute contributed talks running in two parallel sessions and given by 39 speakers. Volume 124 (1994) of the journal *Discrete Mathematics* was a special edition dedicated to this conference; it was edited by Stanley Fiorini and Josef Lauri, and it consisted of 22 selected papers.





Twenty-seven years later we are gathered here for the second such conference organised in the Island of Malta. Although the time-lapse is considerable, the purpose of this conference remains the same as that of the first one: to share and discuss a whole spectrum of topics within the interrelated fields of graph theory and combinatorics. This conference is bringing together almost 200 researchers and mathematicians from 46 different countries spread across all the continents, including 14 invited speakers and a special guest of honour. We have received 150 abstracts for 20-minute contributed talks which will run in four parallel sessions and cover a large variety of areas within graph theory and combinatorics. A special issue of *Discrete Applied Mathematics* dedicated to this conference and containing full-length papers will be published.

This Conference was made possible by the support of the University of Malta and through the financial support we received from our sponsors, whom we thank wholeheartedly. We would also like to thank all those who have helped in some way or another to make this Conference possible. Our final words of thanks go to all of you who are gathered here with us on our tiny Island, and we wish you an enjoyable and successful conference here in Qawra, St Paul's Bay.

Finally, our gratitude goes to Professor Fiorini, without whose invaluable contribution we would not be here today. *Ad multos annos!*

Peter Borg

John Baptist Gauci

Josef Lauri

Irene Sciriha

18 June 2017

Contents

Professor Stanley Fiorini

Short biography

Stanley Fiorini was born in November 1941, so he is now mid-way between his 75$^{\text{th}}$ and 76$^{\text{th}}$ birthdays. In 1963 he started reading Philosophy at Heythrop College when it was still near Oxford and he graduated in 1966. Then he read Mathematics at Oxford starting in 1968 and graduating in 1971. It was here that he first met Robin Wilson who encouraged him to do a PhD in graph theory under his supervision after graduating from Oxford. So, in 1972 he registered as the first PhD student of the Open University, and he submitted his thesis on *The Chromatic Index of Simple Graphs* in 1975. That year he joined the Mathematics Department at the University of Malta, and that was the first time that Graph Theory was introduced in our courses: Josef Lauri was the first student in 1976 to do an MSc at the University of Malta with one of the exam papers being the first one set by Stanley Fiorini on Graph Theory.

In 1977 Stanley and Robin published with Pitman their book *Edge-colourings of Graphs*. From 1979 to 1981 Stanley was again at the Open University, this time as a Staff Tutor at the Bristol Regional Office. Josef Lauri joined him there as a PhD student with the Open University under his supervision. While in Bristol, Stanley also contributed to the writing of the course *TM361: Graphs, Designs and Networks*, mainly in the writing of Units 4 and 6 of the course.

Stanley came back to the University of Malta in 1981 where he remained until his retirement in 2006. This was the time when Graph Theory started to flourish at the University of Malta. In 1990 we held the First Malta Conference on Graph Theory and Combinatorics. Stanley started off both Irene Sciriha and John Baptist Gauci on their first steps in research in Graph Theory, and these went on to do their PhD with Anthony Hilton (who therefore deserves some credit for paternity of Graph Theory in Malta). Stanley also encouraged Peter Borg to do his PhD with Fred Holroyd, also at the Open University. We seem to have an affinity with the Open University because another colleague in our Department, Anton Buhagiar, also obtained his PhD there but in Statistical Mechanics under Oliver Penrose, although Stanley did teach Anton Mathematics when he was at secondary school so there is a link somewhere there too. Stanley was also Head of the Department of Mathematics from 1994 to 2002. Graph Theory is now one of the main streams which our BSc Honours students can choose as a special area in the final two years of their degree. We have had numerous MSc students specialising in Graph Theory, four PhDs and we also have a current PhD student.

And we cannot fail to mention that Stanley Fiorini is one of Maltas foremost Medieval historians, in which area he has published eighty papers and twenty-four books. In November 2006, the University of Malta honoured this polymath by appointing him **Senior Fellow** "in view of his contribution to the University and his ongoing research and scholarship on Maltese History". He is one of only three academics who have been bestowed this honour by the University of Malta.

List of Publications in Mathematics

THESIS

The Chromatic Index of Simple Graphs. Doctoral Thesis, The Open University (U.K.) (1974).

BOOKS

1. *Edge-colourings of Graphs*, Research Notes in Mathematics, No. 16, Pitman, London (1977) (with R.J. Wilson). [Mathematical Reviews (=MR)58#27599].

2. *Selections and Distributions:* Unit 4 in the Open University Course TM361: 'Graphs, Designs and Networks' (1980) (with Course Team).

3. *Planarity and Colouring:* Unit 6 of TM361 (with Course Team).

4. *Graphs and Combinatorics.* (Proceedings of the First Malta Conference) = *Discrete Mathematics*, 124 (1994) (edited with J. Lauri).

PAPERS

1. On the chromatic index of a graph: I, *Cahiers du Centre dEtudes de Recherches Operationnelle*, 15 (1973) 252-262; (with R.J. Wilson). [MR 50#6864a, ZFM 278/05106].

2. On the chromatic index of a graph: II, in: T.P. McDonough and V.C. Mavron (eds.), *Combinatorics. Proceedings of the Fourth British Combinatorial Conference, 1973*, (1974) 37-51; (with R.J. Wilson). [MR 50#6864b, ZFM 297/05110].
3. On the chromatic index of a graph, III: Uniquely edge-colourable graphs, *Quarterly Journal of Mathematics (=QJM) (Oxford)(3)*, 26 (1975) 129-140. [MR51 #7925, ZFM 312/0514].
4. On the chromatic index of outerplanar graphs, *Journal of Combinatorial Theory(B)(=JCT)*, 18 (1975) 35-388. [MR 71#2971, ZFM 273/05107].
5. Some remarks on a paper by Vizing on critical graphs, *Mathematical Proceedings of the Cambridge Philosophical Society (MPPS)*, 77 (1975) 475-483. [MR 51#10146, ZFM 306/05120].
6. On the girth of graphs critical with respect to edge-colourings, *Bulletin of the London Mathematical Society*, 8 (1976) 81-86. [MR 53#13017, ZFM 317/05106].
7. Un grafo cubico, non-planare, unicamente tricolorabile, di vita 5, *Calcolo*, 13 (1976) 105-108. [MR 56#8408, ZFM 339/05104].
8. Edge-colourings of graphs: Some applications, *Proceedings of the Fifth British Combinatorial Conference, 1975*, (1976) 193-202; (with R.J. Wilson). [MR 52 #13461].
9. On small graphs critical with respect to edge-colourings, *Discrete Mathematics(=DM)*, 16 (1976) 109-121; (with L.W. Beineke). [MR 55#2631].
10. On the edge-reconstruction of planar graphs, *MPPS Cambridge*, 83 (1978) 31-35. [MR 58#5313, ZFM 382/05044].
11. A bibliographic survey of edge-colorings, *Journal of Graph Theory(=JGT)*, 2 (1978) 93-106. [MR 58#21754].
12. Counterexamples to two conjectures of Hilton, *JGT*, 2 (1978) 261-264.
13. A theorem on planar graphs with an application to the reconstruction problem: I, *QJM Oxford (2)*, 29 (1978) 353-361. [MR 82d#05083a, ZFM 392/05023].
14. Edge-colourings of graphs, in: L.W. Beineke and R.J. Wilson (eds.), *Selected Topics in Graph Theory*, (1978) 103-126; (with R.J. Wilson).
15. A theorem on planar graphs with an application to the reconstruction problem: II, *Journal of Combinatorics, Information and Systems Sciences*, 3 (1978) 103-119; (with B. Manvel). [MR 82d#05083a].
16. The reconstruction of maximal planar graphs, I: Recognition, *JCT(B)*, 30 (1981) 188-195; (with J. Lauri). [MR 82i#05055a].
17. Edge-reconstruction of 4-connected planar graphs, *JGT*, 6 (1982) 33-42; (with J. Lauri). [MR 83g#05054].
18. On the edge-reconstruction of graphs which triangulate surfaces, *QJM Oxford (2)*, 33 (1982) 191-214; (with J. Lauri). [MR 83h#05065].

19. Hypohamiltonian snarks, in: M. Fiedler (ed.), *Proceedings of the Third Czechoslovak Symposium in Graph Theory, Prague 1982*, (1983) 70-75. [MR 85g#05095].

20. Edge-reconstruction of graphs with topological properties, *Annals of Discrete Mathematics (=ADM)*, 17 (1983) 285-288; (with J. Lauri). [MR 87d#05602].

21. On the crossing number of generalized Petersen graphs, *ADM*, 30 (1986) 225-242. [MR 87k#05073].

22. On the density, chromatic number and chromatic index of a graph, in: F. Mazzocca (ed.), *Atti del Convegno Internazionale di Geometrie Combinatorie, 1988*, (1991) 397-406; (with G. Lofaro and L. Puccio).

23. The chromatic index of graphs: A survey, in: R. Ellul-Micallef and S. Fiorini (eds.), *Collected Papers* (Malta, 1992) 393-419.

24. Map colouring saga, in: J. Schir et al. (eds.), *Liber Amicorum Dr. Albert Ganado*, (Malta, 1994) 121-125.

25. Minimal basis for a vector space with an application to singular graphs, *Graph Theory Notes of New York*, xxxi (1996) 21-24; (with I Sciriha and J. Lauri).

26. On the characteristic polynomial of homeomorphic images of a graph, *DM*, 174 (1997) 293-308; (with I. Sciriha).

27. Necessary and Sufficient Conditions for the Zarankiewicz Conjecture on the Crossing Number, *Graph Theory Notes of New York*, xli (2001) 17-21; (with J.B. Gauci).

28. New results and problems on crossing numbers, in E.M. De Marzi (ed), *Rendiconti del Seminario Matematico di Messina*, Ser. II n. 8 (2002) 29-47; (with J.B. Gauci).

29. The Crossing Number of the Generalised Petersen Graph $P[3k, k]$, *Mathematica Bohemica*, 128/4 (2003) 337-347; (with J.B. Gauci).

30. Trees with greatest nullity, *Linear Algebra and its Applications*, 397 (2005) 245-251; (with I. Gutman and I. Sciriha).

31. k-to-1 functions between complete graphs of even order, *Discrete Mathematics*, 310 (2010) 330-346; (with J.K. Dugdale, J.B. Gauci and A.J.W. Hilton).

Conference Information

The Second Malta Conference in Graph Theory and Combinatorics - 2MCGTC2017
db San Antonio Hotel + Spa, Qawra, St. Paul's Bay, Malta
26 – 30 June 2017

Organised by:
Department of Mathematics, Faculty of Science, University of Malta

Organising Committee:
Peter Borg ⋄ John Baptist Gauci ⋄ Josef Lauri ⋄ Irene Sciriha

Website:
`http://www.um.edu.mt/events/2mcgtc2017/`

e-mail address:
`2mcgtc2017@um.edu.mt`

In honour of:
Stanley Fiorini's 75$^{\text{th}}$ birthday

Invited speakers:
Richard A. Brualdi ⋄ Yair Caro ⋄ Peter Dankelmann ⋄ Patrick W. Fowler, F.R.S. Chris Godsil ⋄ Wilfried Imrich ⋄ Gyula O.H. Katona ⋄ Sandi Klavžar Mikhail Klin ⋄ Imre Leader ⋄ Brendan McKay ⋄ Karen Meagher Raffaele Scapellato ⋄ Andrew Thomason

Special guest:
Robin J. Wilson

Conference Sponsors

Ministry for Finance
Website: http://www.mfin.gov.mt
e-mail: info.mfin@gov.mt

Conventions Malta
Website: www.conventionsmalta.com
e-mail: info@conventionsmalta.com

Ministry for Education and Employment

Website: www.edu.gov.mt

Heritage Malta

Website: http://heritagemalta.org

Norbert Vella Photography
Website: www.facebook.com/norbertvella2
e-mail: norbertvella@outlook.com

Nestlé Malta

Website: www.nestle.com.mt

Ministry for Gozo
Website: http://mgoz.gov.mt
e-mail: gozo@gov.mt

The Programme

Overview

Registration: Sunday 17.00–22.00 and Monday 07.30 onwards

<table>
<tr><th></th><th>Mon</th><th>Tue</th><th>Wed</th><th>Thu</th><th>Fri</th></tr>
<tr><td>08.55–09.00</td><td>Welcome</td><td></td><td></td><td></td><td></td></tr>
<tr><td>09.00–10.00</td><td>Plenary Talk</td><td>Plenary Talk</td><td>Plenary Talk</td><td>Plenary Talk</td><td>Plenary Talk</td></tr>
<tr><td>10.00–11.00</td><td>Plenary Talk</td><td>Plenary Talk</td><td>Plenary Talk</td><td>Plenary Talk</td><td>Plenary Talk</td></tr>
<tr><td>11.00–11.20</td><td>Coffee Break</td><td>Coffee Break</td><td>Coffee Break</td><td>Coffee Break</td><td>Coffee Break</td></tr>
<tr><td>11.20–11.40</td><td rowspan="4">Contributed Talks</td><td rowspan="4">Contributed Talks</td><td rowspan="2">Contributed Talks</td><td rowspan="4">Contributed Talks</td><td rowspan="4">Contributed Talks</td></tr>
<tr><td>11.40–12.00</td></tr>
<tr><td>12.00–12.20</td><td rowspan="2">Lunch Break</td></tr>
<tr><td>12.20–12.40</td></tr>
<tr><td>12.40–14.00</td><td>Lunch Break</td><td>Lunch Break</td><td rowspan="13">Conference Excursion</td><td>Lunch Break</td><td>Lunch Break</td></tr>
<tr><td>14.00–15.00</td><td>Plenary Talk</td><td>Plenary Talk</td><td>Plenary Talk</td><td rowspan="9">Optional Excursion 1</td></tr>
<tr><td>15.00–15.20</td><td>Coffee Break</td><td rowspan="3">Plenary Talk</td><td>Coffee Break</td></tr>
<tr><td>15.20–15.40</td><td rowspan="7">Contributed Talks</td><td rowspan="7">Contributed Talks</td></tr>
<tr><td>15.40–16.00</td></tr>
<tr><td>16.00–16.20</td><td>Coffee Break</td></tr>
<tr><td>16.20–16.40</td><td rowspan="4">Contributed Talks</td></tr>
<tr><td>16.40–17.00</td></tr>
<tr><td>17.00–17.20</td></tr>
<tr><td>17.20–17.40</td></tr>
<tr><td>17.40–18.00</td></tr>
<tr><td>18.00–19.00</td><td></td><td></td><td></td><td></td></tr>
<tr><td>19.00–20.00</td><td>Opening Ceremony</td><td></td><td></td><td rowspan="2">Optional Excursion 2</td></tr>
<tr><td>20.00–</td><td>Welcome Reception</td><td></td><td>Conference Dinner</td></tr>
</table>

Daily

Monday 26 June 2017

08.55–09.00	Welcome			
09.00–10.00	Plenary Talk: *Andrew G. Thomason*			
10.00–11.00	Plenary Talk: *Sandi Klavžar*			
11.00–11.20	Coffee Break			
Venue: *Chair:*	*Rabat* *J. Przybyło*	*Fes* *R. Kalinowski*	*Nador* *J. Muscat*	*Tangier* *E. Năstase*
11.20–11.40	M.C. Golumbic	G. Kiss	R. Fernandes	I. Wanless
11.40–12.00	K. Varga	G. Mazzuoccolo	N. Bebiano	B. De Bruyn
12.00–12.20	G.Y. Katona	E. Kubicka	A. Sali	S. De Winter
12.20–12.40	M. Ellingham	K. Pastuszak	D. Crnkovic	T. Vučičić
12.40–14.00	Lunch Break			
14.00–15.00	Plenary Talk: *Wilfried Imrich*			
15.00–15.20	Coffee Break			
Venue: *Chair:*	*Rabat* *G.Y. Katona*	*Fes* *Z. Tuza*	*Nador* *N. Bebiano*	*Tangier* *G. Hurlbert*
15.20–15.40	G. Simonyi	C. Bujtás	I. Sciriha	J. Pulaj
15.40–16.00	G. Kubicki	M. Pilśniak	S. Pavlíková	P. Borg
16.00–16.20	G. Boruzanlı Ekinci	R. Kalinowski	A. Abiad	V.M. Kamat
16.20–16.40	J. Przybyło	T. Duy Doan	A. Farrugia	F. Ihringer
Venue: *Chair:*	*Rabat* *G. Boruzanlı Ekinci*	*Fes* *G. Gévay*	*Nador* *R. Fernandes*	*Tangier* *P.L. Erdős*
16.40–17.00	N.E. Clarke	T.K. Samuel	H.F. da Cruz	D. Soltész
17.00–17.20	L. Montero	T. Dzido	R.R. Del-Vecchio	Z.L. Nagy
17.20–17.40	Y. Akhtar	E. Năstase	E. Kaya	S.H. Afzali Borujeni
17.40–18.00	S. Stephen	A. Roca	Y. Manoussakis	C. Hernando
18.00–19.00				
19.00–20.00	Opening Ceremony			
20.00–	Welcome Reception			

Tuesday 27 June 2017

09.00–10.00	Plenary Talk: *Imre Leader*			
10.00–11.00	Plenary Talk: *Yair Caro*			
11.00–11.20	Coffee Break			
Venue: *Chair:*	*Rabat* *I. Sciriha*	*Fes* *S. De Winter*	*Nador* *I. Schiermeyer*	*Tangier* *E. Kubicka*
11.20–11.40	R. Wilson	S. Rukavina	M. Frick	A. Nakamoto
11.40–12.00	B. Toft	J. Kokkala	J. de Wet	Y. Asayama
12.00–12.20	J. Lauri	A.H. Bilge	I.A. Goldfeder	J. Sedlar
12.20–12.40	H. Gropp	B. Sahu	A. Asratian	D. Pinto
12.40–14.00	Lunch Break			
14.00–15.00	Plenary Talk: *Brendan McKay*			
15.00–16.00	Plenary Talk: *Mikhail Klin*			
16.00–16.20	Coffee Break			
Venue: *Chair:*	*Rabat* *J.B. Gauci*	*Fes* *A. Malnič*	*Nador* *C. Bujtás*	*Tangier* *A. Abiad*
16.20–16.40	I. Schiermeyer	T. Adachi	H. Gropp	J. Muscat
16.40–17.00	K.J. Asciak	L.K. Jørgensen	G. Gévay	R. Bailey
17.00–17.20	K. Fenech	A. Ramos Rivera	C. Deshpande	S. Furtado
17.20–17.40	R. Lewis	B.K. Sahoo	C. Zarb	O. Çolakoğlu Havare
17.40–18.00	C. Brause	J. Fraser	J. Pavlík	M. Boccia

Wednesday 28 June 2017

09.00–10.00	Plenary Talk: *Richard A. Brualdi*			
10.00–11.00	Plenary Talk: *Patrick W. Fowler*			
11.00–11.20	Coffee Break			
Venue: *Chair:*	*Rabat* *S. Bonvicini*	*Fes* *A. Rădulescu*	*Nador* *P. Borg*	*Tangier* *K.R. Sharaf*
11.20–11.40	J.-G. Caputo	A. Vince	A.J.W. Hilton	R. Simanjuntak
11.40–12.00	D.G. Wang	K. Nishio	A. Kupavskii	G. Greaves
12.00–12.50	Lunch Break			
12.50–	Conference Excursion			

Thursday 29 June 2017

09.00–10.00	Plenary Talk: *Chris Godsil*			
10.00–11.00	Plenary Talk: *Gyula O.H. Katona*			
11.00–11.20	Coffee Break			
Venue: *Chair:*	*Rabat* *L.K. Jørgensen*	*Fes* *A. Farrugia*	*Nador* *C. Zarb*	*Tangier* *A.J.W. Hilton*
11.20–11.40	K.R. Sharaf	P. Hansen	P.L. Erdős	Z. Tuza
11.40–12.00	K. Meslem	M. Borg	G. Rinaldi	B. Patkós
12.00–12.20	P. Šparl	A.D. Maden	S. Bonvicini	D.T. Nagy
12.20–12.40	S. Satake		M. Šajna	J. Barát
12.40–14.00	Lunch Break			
14.00–15.00	Plenary Talk: *Karen Meagher*			
15.00–15.20	Coffee Break			
Venue: *Chair:*	*Rabat* *J. Lauri*	*Fes* *Y. Zelenyuk*	*Nador* *G. Mazzuoccolo*	*Tangier* *F.M. Bhatti*
15.20–15.40	A. Malnič	G. Hurlbert	H. Furmańczyk	F. Lazebnik
15.40–16.00	J. Šiagiová	L.F. Papp	P. Repolusk	M. Isaev
16.00–16.20	P. Leopardi	A. Rădulescu	R.M. Casablanca	D. Knop
16.20–16.40	D. Merkle	W.H.T. Wong	S.S. Zemljič	T. Toufar
Venue: *Chair:*	*Rabat* *K. Asciak*	*Fes* *S. Rukavina*	*Nador* *G. Rinaldi*	*Tangier* *V. Liskovets*
16.40–17.00	N. Tratnik	R.M. Falcón	L.A. Dosal-Trujillo	J. Tuite
17.00–17.20	A. Behmaram	T.G. Marbach	J.A. Fresán Figueroa	C. Justel
17.20–17.40	A. Taranenko	V.S. Nittoor	K.L. Patra	Y. Gu
17.40–18.00	S.W. Saputro	E. Saygı	D.A. Jaume	
18.00–20.00				
20.00–	Conference Dinner			

Friday 30 June 2017

09.00–10.00	Plenary Talk: *Peter Dankelmann*			
10.00–11.00	Plenary Talk: *Raffaele Scapellato*			
11.00–11.20	Coffee Break			
Venue: *Chair:*	*Rabat* *M. Isaev*	*Fes* *A. Taranenko*	*Nador* *F. Lazebnik*	
11.20–11.40	V. Liskovets	D. Miklós	F.M. Bhatti	
11.40–12.00	P. Codara	W. Kubiak	M. Vizer	
12.00–12.20	H. Acan	T. Masařk	A. Angeleska	
12.20–12.40	Y. Zelenyuk	O. Hudry		
12.40–14.00	Lunch Break			
14.00–18.00	Optional Excursion 1			
18.00–19.00				
19.00–	Optional Excursion 2			

Abstracts of Plenary Talks

Alternating Sign Matrices and Hypermatrices

Wednesday
09.00-10.00

Richard A. Brualdi

University of Wisconsin-Madison, U.S.A.

brualdi@math.wisc.edu

(*joint work with* Geir Dahl and Michael Schroeder)

Alternating Sign Matrices (ASMs) are square $(0, \pm 1)$-matrices such that, ignoring 0's, the +1's and −1's in each row and column alternate beginning and ending with a +1. Permutation matrices are the ASMs without any −1's. We shall discuss the origins and properties of ASMs. There is a partial order on permutation matrices, the so-called Bruhat order, which extends in a very natural and surprising way to ASMs. This partial order is ranked and has many interesting properties.

There are hypermatrix generalizations of permutation matrices which lead to hypermatrix generalizations of ASMs and latin squares.

Repetitions in the degree sequence of a graph

Tuesday
10.00-11.00

Yair Caro

University of Haifa-Oranim, Israel

yacaro@kvgeva.org.il

A basic fact in graph theory is that every graph G on $n \geq 2$ vertices has at least two vertices of the same degree. In my lecture I will focus on two (the second and the fourth)of the five areas listed below (for the completeness of the picture).

There are basically five areas of research that have been studied under the umbrella "repetition of degrees".

1. *r-uniform hypergraphs* — where this repetition property of having at least two vertices of the same degree no longer holds.

It is known that for $r \geq 3$ and $n \geq r+3$ there exist r-uniform hypergraphs on n vertices with all vertex degrees distinct.

This result appears in [1] (for recent results see [2]).

2. *The repetition number of a graph, rep(G)* — this is the number of times the most frequent value appears in the degree sequence of a graph. Two main directions have been studied:

   (a) The connection between $rep(G)$ and the independence number of a graph $\alpha(G)$ in K_r-free graphs for $r \geq 3$ studied by Erdős and his co-authors [3] and nearly completed by Bollobás and Scott [4,5].

   Only one case remains open after nearly 20 years.

   (b) The connection between $rep(G)$ and the minimum degree , average degree and maximum degree denoted respectively $\delta = \delta(G)$, $d = d(G)$, and $\Delta = \Delta(G)$ studied by Caro and West [6] .

   I will exhibit extremal graphs belonging to natural families of graphs such as trees, maximal outerplanar graphs, maximal planar graphs in which the general lower bound $rep(G) \geq \frac{n}{2d-2\delta+1}$ is attained [6].

   A disturbing open problem remains with regards the repetition number of line graphs, which I shall discuss in the lecture.

3. *Deleting vertices from a graph G to get an induced subgraph H with rep(H) ≥ k* — we denote by $f_k(G)$ the minimum number of vertices needed to be deleted from a graph G to obtain a graph H with $rep(H) \geq k$ (providing G is not too small) .

   Clearly $f_2(G) = 0$ is the basic fact stated above. While a priori it is not at all clear that $f_k(G)$ is independent of the order of G, $f_k(G)$ has been proven to be finite [7]. More precisely $f_k(G) \leq (8k)^k$ for $|G|$ sufficiently large , yet many questions remain open.

4. *Deleting vertices from a graph G to get an induced subgraph H with at least k vertices realizing Δ(H)* — we denote by $g_k(G)$ the minimum number of vertices needed to be deleted from G to obtain an induced subgraph H with at least k vertices realizing $\Delta(H)$ (provided G is not too small) .

   Unlike the situation in $f_k(G)$ which is finite, already $g_2(G)$ can be as large as $(1+o(1))\sqrt{2|G|}$ and $g_3(G) \leq 43\sqrt{|G|}$ is a non-trivial result of [8]. I shall discuss the recent progress concerning $g_2(G)$.

   However major problems remain widely open.

5. *The notion of spread* — for a sequence $A = \{a_1, a_2, \ldots, a_n\}$ the spread of A, denoted $sp(A) = \max\{a_j : a_j \in A\} - \min\{a_i : a_i \in A\}$.

   Clearly if G is a graph on $n \geq 2$ vertices there are at least two vertices v_1, v_2 such that $sp(A) = sp\{deg(v_1), deg(v_2)\} = 0$ which is the basic fact we started from.

Generalizing this elementary fact it is proved in [9] that for every k, $0 \leq k \leq n-2$ a graph G on n vertices contains $k+2$ vertices $v_1, \ldots, v_{k+2}$ such that $sp\{deg(v_1), \ldots, deg(v_{k+2})\} \leq k$.

The proof is based upon the celebrated Erdős-Gallai theorem characterizing degree sequences.

If time permits I will show a recently discovered charming elementary proof of this results avoiding the Erdős-Gallai theorem, and discuss possible extensions of problems stated under the notion of $rep(G)$ to the setting of spread.

References:

[1] A. Gyarfas, M. S. Jacobson, L. Kinch, J. Lehel & R. H. Schelp, Irregularity strength of uniform hypergraphs, *J. Combin. Math. Combin. Comput.* **11** (1992) 161–172.

[2] P. Balister, B. Bollobás, J. Lehel, & M. Morayne, Random Hypergraph Irregularity, *IAM J. Discrete Math.* **30** (2016) 465–473.

[3] P.Erdős, R. Faudree, T.J. Reid, R. Schelp & W. Staton, Degree sequence and independence in K(4)-free graphs, *Discrete Mathematics* **141** (1995) 285–290.

[4] B. Bollobás & A.D. Scott, Independent sets and repeated degrees, *Discrete Mathematics* **170** (1997) 41–49.

[5] B. Bollobás, Degree multiplicities and independent sets in K_4-free graphs, *Discrete Mathematics* **158** (1996) 27–35

[6] Y. Caro & D.B. West, Repetition number of graphs, *Electronic Journal of Combinatorics* **16** (2009) #R7

[7] Y. Caro, A. Shapira & R. Yuster, Forcing k-repetitions in degree sequences, *Electronic Journal of Combinatorics* **21** (2014) #R24

[8] Y. Caro & R. Yuster, Large induced subgraphs with equated maximum degree, *Discrete Mathematics* **310** (2010) 742–747.

[9] P. Erdős, G. Chen, C.C. Rousseau & R.H. Schelp, Ramsey Problems Involving Degrees in Edge-colored Complete Graphs of Vertices Belonging to Monochromatic Subgraphs, *European Journal of Combinatorics* **14** (1993) 183–189.

Friday
09.00-10.00

The Diameter of Graphs and Digraphs

Peter Dankelmann

University of Johannesburg, South Africa

pdankelmann@uj.ac.za

Distance based graph invariants have been studied almost since the inception of graph theory, and among those the diameter, defined as the largest of the distances between all pairs of vertices in a connected graph or strong digraph, has received much attention. In this talk we present some old but possibly not well-known results on the diameter of graphs, as well as several recent results. A special focus will be on results on the diameter of digraphs, a topic which has received much less attention in the literature than the diameter of graphs.

The first part of our talk is on bounds relating the diameter to other graph invariants. Most of these bounds have been shown to hold for undirected graphs, and we discuss possible generalisations of these bounds to digraphs. We show that some bounds that hold for connected graphs, but not for all strong digraphs, extend to Eulerian digraphs, a large class of digraphs containing all graphs. In the second part of our talk we present bounds on the diameter of strong orientations of graphs.

Wednesday
10.00-11.00

Graph theoretical models and molecular currents

Patrick Fowler

University of Sheffield, UK

P.W.Fowler@sheffield.ac.uk

(*joint work with* Martha Borg, Joseph Clarke, Wendy Myrvold, Barry Pickup and Irene Sciriha)

Currents within and through molecules are of interest in chemistry and physics for at least two reasons: circulations induced by a magnetic field (ring currents) are related to experimental NMR signatures of aromatic molecules and the question of how to define aromaticity; ballistic currents induced by potential differences are related to molecular electronics. In both areas, calculations with sophisticated techniques can give valuable information on individual systems, but there is room for simpler models that can describe behaviour of whole families of molecules and devices. Many of the most appealing models are based on graph theory, making applications of graph spectra to ballistic conduction and perfect matchings to ring current. This talk describes some recent work in both areas.

Quantum Walks on Graphs

Thursday
09.00-10.00

Chris Godsil

University of Waterloo, Canada

cgodsil@uwaterloo.ca

Quantum walks are analogs of classical random walks defined by physicists. They underly important algorithms in quantum computing (Grover search, for example) and can also be used to provide an implementation of a quantum computer. Quantum walks come in two flavours—continuous and discrete—and in both cases they are usually defined in terms of an underlying graph. (In the discrete case extra structure may be needed, for example, it may be necessary to specify a 1-factorization or an embedding of the underlying graph.)

The behaviour and properties of these walks can be successfully analysed using standard tools from algebraic graph theory and number theory. In one direction, our work has provided limits on what can be achieved using quantum walks. However these walks also give rise to interesting new graph invariants. My talk will provide an overview of our progress on this topic, and the questions that remain.

On the direct product of finite and infinite graphs

Monday
14.00-15.00

Wilfried Imrich

Montanuniversität Leoben, Austria

wilfried.imrich@unileoben.ac.at

This talk presents recent results about direct products of finite and infinite directed graphs. For finite graphs prime factorizations always exist, and often they are unique. We present new classes of graphs with unique prime factorizations and algorithms to compute them in polynomial time. On the way we also characterize the structure of the automorphism group of such products and investigate various properties, such as vertex- and edge-transitivity.

In the case of infinite graphs prime factorizations need not exist, but if they do, they are unique under certain thinness and connectedness conditions and allow unexpected conclusions about their groups.

Thursday
10.00-11.00

A general 2-part Erdős-Ko-Rado theorem

Gyula O. H. Katona

MTA Rényi Institute, Hungary

ohkatona@renyi.hu

A two-part extension of the famous Erdős-Ko-Rado Theorem is proved. The underlying set is partitioned into X_1 and X_2. Some positive integers $k_i, \ell_i (1 \le i \le m)$ are given. We prove that if $\mathcal{F}$ is an intersecting family containing members F such that $|F \cap X_1| = k_i, |F \cap X_2| = \ell_i$ holds for one of the values $i(1 \le i \le m)$ then $|\mathcal{F}|$ cannot exceed the size of the largest subfamily containing one element. The statement was known for the case $m = 2$ as a result of Frankl.

Monday
10.00-11.00

Packing Chromatic Number

Sandi Klavžar

University of Ljubljana & University of Maribor, Slovenia

sandi.klavzar@fmf.uni-lj.si

The packing chromatic number $\chi_\rho(G)$ of a graph G is the smallest integer k such that the vertex set of G can be partitioned into sets $\Pi_1, \dots, \Pi_k$, where Π_i, $i \in [k]$, is an i-packing. This concept was introduced in [7], given the present name in [2], and extensively studied afterwards. The packing chromatic number is intrinsically difficult, determining χ_ρ is NP-complete even when restricted to trees [5].

In the first part of the talk a brief survey on the packing chromatic number will be given including the very recent progress [8]. Afterwards the problem whether there exists an absolute constant M, such that $\chi_\rho(G) \le M$ holds for any subcubic graph G will be discussed in detail. The recent progress on this problem is enviable: [1, 3, 4, 6].

References:

[1] J. Balogh, A. Kostochka, X. Liu, Packing chromatic number of subcubic graphs, arXiv:1703.09873v2 [math.CO], March 30, 2017.

[2] B. Brešar, S. Klavžar, D.F. Rall, On the packing chromatic number of Cartesian products, hexagonal lattice, and trees, *Discrete Appl. Math.* **155** (2007) 2303–2311.

[3] B. Brešar, S. Klavžar, D.F. Rall, K. Wash, Packing chromatic number under local changes in a graph, *Discrete Math.* **340** (2017) 1110–1115.

[4] B. Brešar, S. Klavžar, D.F. Rall, K. Wash, Packing chromatic number, $(1, 1, 2, 2)$-colorings, and characterizing the Petersen graph, *Aequationes Math.* **91** (2017) 169–184.

[5] J. Fiala, P.A. Golovach, Complexity of the packing coloring problem for trees, *Discrete Appl. Math.* **158** (2010) 771–778.

[6] N. Gastineau, O. Togni, S-packing colorings of cubic graphs, *Discrete Math.* **339** (2016) 2461–2470.

[7] W. Goddard, S.M. Hedetniemi, S.T. Hedetniemi, J.M. Harris, D.F. Rall, Broadcast chromatic numbers of graphs, *Ars Combin.* **86** (2008) 33–49.

[8] B. Martin, F. Raimondi, T. Chen, J. Martin, The packing chromatic number of the infinite square lattice is between 13 and 15, *Discrete Appl. Math.* **225** (2017) 136–142.

Introduction to total graph coherent configurations

Tuesday
15.00-16.00

Mikhail Klin

Ben-Gurion University and Matej Bel University, Israel

klin@cs.bgu.ac.il

(*joint work with* Leif Jørgensen and Matan Ziv-Av)

Starting from a given connected regular undirected graph Γ, its total graph $\mathcal{T}(\Gamma)$ is considered. The coherent closure (in the sense of Weisfeiler-Leman) is constructed. Then we are looking for nice non-trivial mergings of this coherent closure, i.e. those which have a small rank or an interesting automorphism group. In this way we construct from the graph Γ a much larger combinatorial structure with symmetry, which may be expressed in diverse terms.

The results by MZA for two classical infinite classes of rank 3 graphs (2009) showed that just for two members of one of these classes, namely for triangular graphs $T(5)$ and $T(7)$ really interesting mergings appear in our construction.

Using computer algebra packages GAP and COCO, we investigated two exceptional Higmanian rank 5 association schemes on 40 points which appear from $T(5)$, as well as exceptional Zara graph on 126 vertices, stemming from $T(7)$. Links with diverse nice intermediate structures, like Witt desigh W_{24}, hermitian unital on 28 points, generalized hexagon $H(2)$, Möbius-Kantor graph, etc. will be briefly mentioned.

The results for $T(5)$ fit to another project, related to the complements of Moore graphs, to be discussed by LJ.

Tuesday
09.00-10.00

Tiling in High Dimensions

Imre Leader

University of Cambridge, UK

leader@dpmms.cam.ac.uk

Suppose that we have a tile T, meaning a finite subset of the integer grid $\mathbb{Z}^n$ for some n. It may or may not tile $\mathbb{Z}^n$, in the sense that we can partition $\mathbb{Z}^n$ into copies of T. We show, however, that T does tile $\mathbb{Z}^d$ for some d. This confirms a conjecture of Chalcraft.

Tuesday
14.00-15.00

Graph generation and Ramsey numbers

Brendan McKay

Australian National University, Australia

Brendan.McKay@anu.edu.au

(*joint work with* Vigleik Angeltveit)

We will discuss some advances in the practice of graph generation. As examples, we will show how several bounds on small Ramsey numbers can be improved, including upper bounds on $R(5,5)$ and $R(4,6)$.

Thursday
14.00-15.00

Cocliques in Derangement Graphs

Karen Meagher

University of Regina, Canada

karen.meagher@uregina.ca

The *derangement graph* for a group is a Cayley graph for a group G with connection set the set of all derangements in G (these are the elements with no fixed points). The eigenvalues of the derangement graph can be calculated using the irreducible characters of the group. The eigenvalues can give information about the graph, I am particularly interested in applying Hoffman's ratio bound to bound the size of the cocliques in the derangement graph. This bound can also be used to obtain information about the structure of the maximum cocliques. I will present a few conjectures about the structure of the cocliques. This work is attempting to find a version of the Erdős-Ko-Rado theorem for permutations.

Symmetry in graphs and digraphs

Friday
10.00-11.00

Raffaele Scapellato

Politecnico di Milano, Milano, Italy

raffaele.scapellato@polimi.it

We shall present a survey about various well known and less known results concerning graph symmetry. Whenever possible, the corresponding facts and problems in the directed case will be discussed, too. The standard way to introduce and study symmetry in graphs is of course through the concept of an automorphism, so that permutation groups (and sometimes abstract groups, like in the classic Frucht theorem) will have a prominent role. In particular, vertex-transitive graphs (and digraphs) can be obtained from the action of a permutation group. Important special cases like Cayley graphs, arc-symmetric graphs, distance-transitive graphs and so forth will also be discussed, together with their combinatorial counterparts (e.g., distance-regular graphs for the last names case). On the other hand, symmetry can be studied from different standpoints, giving rise to larger permutation groups. A typical instance is similarity of vertices: when two vertices give rise to isomorphic graphs with their removal, this doesn't imply per se that there is an automorphism taking one into another. Several other examples will be illustrated.

List colourings of hypergraphs

Monday
09.00-10.00

Andrew Thomason

University of Cambridge, UK

a.g.thomason@dpmms.cam.ac.uk

(*joint work with* Ares Meroueh)

Suppose we have a (large) palette of colours, and we assign to each vertex v of a graph G some subset $L(v)$ of these. A list colouring of G with lists L is then a choice of colour $c(v)$ for each vertex v such that $c(v)$ is in $L(v)$ and the colouring c is a proper colouring — that is, no edge has both its vertices the same colour. Ordinary graph colouring is a special case of this in which all the lists $L(v)$ are the same. Intuitively this seems like the hardest case but in fact it can be harder to colour when the lists are different. Indeed Alon proved that, for any G, there are lists as large as $\log d$ for which no colouring is possible, where d is the average degree (even if G is bipartite).

An r-uniform hypergraph, or r-graph, has edges which are r-sets of the vertex set, so a graph is just a 2-graph. The notion of list colouring extends immediately to r-graphs: we just ask that for no edge does c assign the same colour to each of its r vertices. Alon's theorem was extended recently to r-graphs by Saxton and myself via the method of containers. But it seems that containers do not tell the full story here. We describe how the notion of preference orders can provide both efficient colouring algorithms and, complementarily, good lower bounds on list colourings of hypergraphs. This gives complete information in the cases $r = 2$ and $r = 3$ but for $r \geq 4$ the situation becomes more interesting and remains unresolved.

Abstracts of Contributed Talks

An application of Hoffman graphs for spectral characterizations of graphs

Monday
16.00-16.20

Aida Abiad

Maastricht University, Netherlands

A.AbiadMonge@maastrichtuniversity.nl

(*joint work with* Jack H. Koolen and Qianqian Yang)

In this paper, we present the first application of Hoffman graphs for spectral characterizations of graphs. In particular, we show that the 2-clique extension of the $(t+1)\times(t+1)$-grid is determined by its spectrum when t is large enough. This result will help to show that the Grassmann graph $J_2(2D, D)$ is determined by its intersection numbers as a distance regular graph, if D is large enough.

Formation of a giant component in the intersection graph of a random chord diagram

Friday
12.00-12.20

Huseyin Acan

Rutgers University, USA

huseyin.acan@rutgers.edu

(*joint work with* Boris Pittel)

A chord diagram of size n is a pairing of $2n$ points. When the points are placed on a circle, this gives n chords. The intersection graph of a chord diagram D is formed by taking the chords of D as the vertices of the graph and creating an edge between two vertices if and only if the corresponding chords cross each other.

Let $H_{n,m}$ denote a uniformly random chord intersection graph with n vertices and m edges. We study the largest component of $H_{n,m}$ for $m = O(n \log n)$. In particular, when $m/(n \log n)$ tends to a limit in $(0, 2/\pi^2)$, we show that the largest component contains almost all the edges and a positive fraction of all the vertices of $H_{n,m}$. On the other hand, when $m \leq n/14$, the size of the largest component is $O(\log n)$.

Thresholds for the appearance of giant components are well studied for various random graph models, most famously for Erdős-Rényi graphs. In the case of a random chord intersection graph, it is not known whether or not there is a threshold. However, if there is such a threshold, our results imply that it must be of order $\Omega(n)$ and $O(n \log n)$.

Reference:

[1] H. Acan and Boris Pittel, Formation of a giant component in the intersection graph of a random chord diagram, *J. Combin. Theory Ser. B* **125** (2017) 33–79.

Tuesday
16.20-16.40

Vertex transitive Kähler graphs whose adjacency operators are commutative

Toshiaki Adachi

Nagoya Institute of Technology, Japan

adachi@nitech.ac.jp

(*joint work with* Guan-Yuan Chen)

In geometry, graphs are considered as discrete models of Riemannian manifolds. When we study Riemannian manifolds we frequently consider some geometric structure on them, complex structures, contact structures and so on. The presenter is hence interested in giving discrete models which inherit geometric structures. He introduced the notion of Kähler graphs in [1]. A Kähler graph is a compound of two graphs having common sets of vertices. More precisely, a simple graph $G = (V, E)$ is said to be *Kähler* if the set E of edges is divided into two disjoint subsets $E^{(p)}$, $E^{(a)}$ and each of two graphs $G^{(p)} = (V, E^{(p)})$, $G^{(a)} = (V, E^{(a)})$ does not have hairs. Here, a hair is an edge one of whose ends is of degree one.

Since some geometric structures induce closed 2-forms, which are also called magnetic fields, and they define trajectories corresponding to geodesics, his idea is to give "curved" paths on graphs because paths on graphs correspond to geodesics. For a pair (p, q) of relatively prime positive integers, we say that a $(p+q)$-step path on G is a (p, q)-*primitive bicolored path* if it is a p-step path on $G^{(p)}$ followed by a q-step path on $G^{(a)}$. A chain of such paths is said to be a (p, q)-bicolored path. We consider paths on $G^{(p)}$ as correspondences of geodesics. Under the influence of a magnetic filed we consider that they are bended and turn to bicolored paths whose first p-step are given paths on $G^{(p)}$.

Among bicolored paths, the simplest one is a $(1, 1)$-bicolored path. If we denote the adjacency operators of $G^{(p)}, G^{(a)}$ by $A^{(p)}, A^{(a)}$, respectively, we find that the generating operator of the random walk by $(1, 1)$-bicolored paths is $A^{(p)}A^{(a)}$. In this talk, considering Laplacians of Kähler graphs, we explain a correspondence between Kähler graphs and Kähler manifolds. We give a condition on the cardinality of the set of vertices and on the degrees of two kinds of graphs to construct vertex-transitive Kähler graphs whose two kinds of adjacency operators are commutative. Such graphs are considered to correspond to homogeneous manifolds.

References:

[1] T. Adachi, A discrete model for Kähler magnetic fields on a complex hyperbolic space, in: K. Sekigawa *et al.* (eds.), *Trends in Differential Geometry, Complex Analysis and Mathematical Physics*, World Scientific (2009) 1–9.

[2] T. Yaermaimaiti & T. Adachi, Isospectral Kähler graphs, *Kodai Math. J.* **38** (2015) 560–580.

Investigating posets via their maximal chains: a Sperner type approach

Monday
17.20-17.40

Seyed Hadi Afzali Borujeni

IPM, Iran

hadiafzali59@yahoo.com

(*joint work with* Nathan Bowler)

Any poset (partially ordered set) is determined by the set of subsets appearing as maximal chains. Now imagine that we are not given the maximal chains but just some information like the number of them in the poset, the number of them going through each element of the poset, or the size of the intersection of a certain subset with each maximal chain. What can we say about the poset in this case?

A classical result in extremal set theory known as Sperner's theorem states that the size of any antichain in the boolean lattice of all subsets of $[n] = \{1, \ldots, n\}$ is at most $\binom{n}{\lfloor \frac{n}{2} \rfloor} = \binom{n}{\lceil \frac{n}{2} \rceil}$. Since then numerous extensions and applications of this result have been found, so that modern extremal set theory is indebted for much of its development to it. Assuming only combinatorial information about numbers of maximal chains we can prove new results extending this theorem to more general posets.

In this talk, after taking a quick tour on the history, we will review our results.

Monday
17.20-17.40

Baranyai's Theorem for Completeness of Bipartite Induced Subhypergraph

Yasmeen Akhtar

Institute of Statistical Science, Academia Sinica, Taiwan

yasmeen@stat.sinica.edu.tw

In this paper, we follow the footstep of the famous Baranyai's theorem from forty years ago. We investigate an approach to equitably and uniformly color hyperedges in a 3-uniform complete tripartite hypergraph with multiple hyperedges, so that each factor corresponding to such a coloring is connected and the sub-hypergraph induced by two sides is in fact completely bipartite. We start by introducing a transformation that maps an almost V_i-regular and almost (V_i, V_j)-co-regular 3-uniform complete tripartite hypergraph to a 3-uniform tripartite balanced hypergraph with a maximum degree. Then we prove that such a hypergraph admits an equitable and uniform k-coloring, and that each corresponding factor hypergraph is a connected hypergraph.

References:

[1] Z. Baranyai, The edge-coloring of complete hypergraphs I, *Journal of Combinatorial Theory*, Series B **26** (1979) 276–294.

[2] Z. Baranyai, La coloration des arêtes des hypergraphes complets II, *Problèmes Combinatoires et Théorie des Graphes*, Coll. C.N.R.S. Orsay 1976, (Bermond, Fournier, Las Vergnas, Sotteau), C.N.R.S., Paris (1978) 19–22.

[3] C. Berge, *Hypergraphs-Combinatorics of finite sets* (first edition) North-Holland Mathematical Library 45, Elsevier Science Publishers B.V., The Netherlands (1989).

[4] M. A. Bahmanian, Connected Baranyais theorem, *Combinatorica* **34** (2014) 129–138.

Friday
12.00-12.20

Coherent Graph Partitions

Angela Angeleska

The University of Tampa, USA

aangeleska@ut.edu

(*joint work with* Zoran Nikoloski)

The main motivation for our study of network partitions was the sequence clustering problem in the next generation sequencing. We define a coherent partition of a graph G as a vertex partition of G that results in a vertex partition in $\overline{G}$ (the complement of G) that is composed of disconnected subgraphs only. Furthermore, we introduce the notion of coherent number of a graph G, defined as a cardinality of the minimum edge cut over all coherent partitions of G. An optimal coherent partition is a coherent partition that realizes the coherent number of G. Coherent partitions (coherent numbers) are studied in connection to clique and biclique partitions (clique and biclique cover numbers). We also investigate the complexity of the problem of finding optimal coherent partitions, which is polynomial for trees, but NP in general.

Generating theorem of even triangulation on the Klein bottle

Tuesday
11.40-12.00

Yoshihiro Asayama

Yokohama National University, Japan

asayama-yoshihiro-yd@ynu.jp

(*joint work with* Naoki Matsumoto and Atsuhiro Nakamoto)

We define two reductions, a 4-contraction and a twin-contraction, for even triangulations on a surface. It is well known that these reductions preserve some properties of graphs. The complete lists of minimal even triangulations for the sphere, the projective plane and the torus with respect to these reductions have already determined [1,2,3]. In this talk we present the complete list of minimal even triangulations of the Klein bottle, and we discuss some applications.

References:

[1] V. Batagelj, Inductive definition of two restricted classes of triangulations, *Discrete Math.* **52** (1984) 113–121.

[2] N. Matsumoto, A. Nakamoto and T. Yamaguchi, Generating even triangulation on the torus, submitted.

[3] Y. Suzuki and T. Watanabe, Generating even triangulations of the projective plane, *J. Graph Theory* **56** (2007) 333–349.

Tuesday
16.40-17.00

Graphs with large vertex and edge reconstruction number

Kevin Joseph Asciak

University of Malta, Malta

kevin.j.asciak@um.edu.mt

The reconstruction problem is considered to be one of the most important unsolved problems in graph theory. Proposed in 1942, the Graph Reconstruction Conjecture of a graph G states that every simple, finite and undirected graph with three or more vertices can be reconstructed up to isomorphism to the original graph from the collection of all unlabelled vertex-deleted subgraphs of G.

Related to this conjecture, Harary and Plantholt came up with the idea of the reconstruction number - the minimum number of vertex-deleted subgraphs required in order to identify a graph up to isomorphism. The edge-reconstruction number of a graph is analogously defined.

Myrvold and independently Bollobás proved that almost all graphs have reconstruction numbers equal to 3. An analogous result for edges showed that almost every graph has an edge-reconstruction number of 2.

We shall look into some results obtained in recent years which dealt with graphs that have large vertex and edge reconstruction numbers.

Tuesday
12.20-12.40

Localization theorems on Hamilton cycles

Armen Asratian

Linköping University, Sweden

armen.asratian@liu.se

The classical global criteria for the existence of Hamilton cycles and paths only apply to the graphs with large edge density and small diameter.

In a series of papers (see, for example [1]-[6]) we have developed some local criteria for the existence of Hamilton cycles in a connected graph, which are analogues of the global criteria due to Dirac, Ore and others. The idea was to show that the global concept of hamiltonicity can, under rather general conditions, be captured by local phenomena, using the structure of balls of small radii. (For a vertex u of a graph G and an integer $r \geq 1$, the ball of of radius r centered at u is the subgraph induced by the set of all vertices of G whose distance from u does not exceed r.)

This local approach gives the possibility to find new classes of graphs with Hamilton cycles which, in particular, also contain infinite subclasses of graphs with small edge density and large diameter.

I shall give a review of this topic and present some new results.

References:

[1] A.S. Asratian, New local conditions for a graph to be hamiltonian, *Graphs and Combinatorics* **22** (2006) 153–160.

[2] A.S. Asratian, Some properties of graphs with local Ore conditions. *Ars Combinatoria*, **41** (1995) 907–106.

[3] A.S. Asratian, H.J.Broersma, J.van den Heuvel and H.J.Veldman, On graphs satisfying a local Ore - type condition, *J. Graph Theory*, **21** (1996) 1–10.

[4] A.S. Asratian and N.K.Khachatrian, Some localization theorems on hamiltonian circuits, *J. of Combinatorial Theory Series B* **49** (1990) 287–294.

[5] A.S. Asratian and N.K. Khachatrian, On the local nature of some classical theorems on Hamilton cycles, *Australasian J. Combinatorics*, **38** (2007) 77–86.

[6] A.S. Asratian and N. Oksimets, Graphs with hamiltonian balls, *Australasian J. Combinatorics*, **17** (1998) 185–198.

Orthogonal matrices with zero diagonal

Tuesday
16.40-17.00

Robert Bailey

Grenfell Campus, Memorial University, Canada

rbailey@grenfell.mun.ca

(*joint work with* Robert Craigen)

A square real matrix M is orthogonal if and only if $MM^T = cI$ for some non-zero scalar c (or, equivalently, its inverse is a scalar multiple of its transpose). In this talk, we consider orthogonal matrices whose diagonal entries are zero and off-diagonal entries are non-zero. We show how to construct such a matrix for any even order, and obtain infinite families of such matrices of odd order, using techniques old and new. Finally, we apply our results to determine the minimum number of distinct eigenvalues of matrices associated with certain bipartite graphs, and consider the related notion of orthogonal matrices with partially-zero diagonal.

Thursday
12.20-12.40

Extremal K_5-minor-free graphs with fixed girth

János Barát

MTA-ELTE Geometric and Algebraic Combinatorics Research Group
and
University of Pannonia, Hungary

barat@cs.elte.hu

(*joint work with* David R. Wood)

A typical question of extremal graph theory is the following. *What is the maximum number of edges in a graph that belongs to a certain class?* We study the classes of K_5-minor-free graphs with fixed girth g. We show a general result for an infinite number of values $g = 4k$. On the other hand, we give a detailed argument for the most interesting case, when the girth is 5. There is an infinite class of extremal graphs in this case. We indicate that the same principles should work for other values of the girth, but what about K_6-minor-free graphs?

Monday
11.40-12.00

Real spectra in non-Hermitian operators

Natália Bebiano

University of Coimbra, Portugal

bebiano@mat.uc.pt

The spectral analysis of non-Hermitian unbounded operators appearing in quantum physics is our main concern. We show that the so-called equation of motion method, which is well known from the treatment of Hermitian operators, is also useful to obtain the explicit form of the eigenfunctions and eigenvalues of these non-Hermitian operators. We also demonstrate that the considered operators can be diagonalized when they are expressed in terms of certain conveniently constructed operators. We show that their eigenfunctions constitute complete systems, but do not form Riesz bases.

Counting Matchings in a class of Fullerene Graphs

Thursday
17.00-17.20

Afshin Behmaram

University of Tabriz, I.R. Iran

behmaram@tabrizu.ac.ir

Classical fullerene graphs have been intensely researched since the discovery of buckminsterfullerene in the fundamental paper [5], which appeared in 1985. This paper gave rise to the whole new area of fullerene science.

A **fullerene graph** is a cubic, planar, 3-connected graph with only pentagonal and hexagonal faces. A connected 3-regular planar graph $G = (V, E)$ is called an m-**generalized fullerene** [1] if exactly two of its faces are m-gons and all other faces are pentagons and/or hexagons. (We also count the outer (unbounded) face of G.) In the rest of the paper we only consider $m \geq 3$; note that for $m = 5, 6$ an m-generalized fullerene graph is a classical fullerene graph. As for the classical fullerenes it is easy to show that the number of pentagons is fixed, while the number of hexagons is not determined. The smallest m-generalized fullerene has $4m$ vertices and no hexagonal faces. Such graphs are sometimes called m-**barrels**. They have two m-gons and $2m$ pentagons and they can be elongated by inserting $k \geq 0$ layers of m hexagons between two half-barrels. The elongated barrels are one of the main subjects of this paper, since their highly symmetric structure allows for obtaining good bounds and even exact results on the number of perfect matchings in them.

A **matching** M in a graph G is a collection of edges of G such that no two edges of M share a vertex. If every vertex of G is incident to an edge of M, the matching M is called **perfect**. Perfect matchings have played an important role in chemical graph theory, in particular for benzenoid graphs, where their number correlates with the compound's stability. Let $\Phi(G)$ be the number of perfect matchings in G.

Now, we describe a special family of m-generalized fullerene. An **elongated barrel** $F(m, k)$ is obtained from the corresponding barrel by inserting $k \geq 0$ layers (or rings) of m hexagons between two halves of the barrel. For $m = 5$ and $m = 6$ we obtain classical fullerene nanotubes. Most of the nanotube properties are also preserved by elongated barrels. Note that $F(m, k)$ has $n = 2m(k + 2)$ vertices.

The problem of hamiltonicity of fullerene graphs had been open for a long time. There were several partial results, until this special case of Barnette's conjecture was settled by Kardoš, who provided a computer-assisted proof [4].

Theorem 1. *For all natural numbers $m \geq 3$ and k, $F(m, k)$ is Hamiltonian.*

The existence of Hamiltonian cycles has several consequences important for matchings-related properties of elongated barrell.

Theorem 2. *$F(m, k)$ has at least three different perfect matchings. Moreover, each edge of $F(m, k)$ is contained in some perfect matching of $F(m, k)$.*

Theorem 3. *The number of perfect matchings in $F(m,k)$ is bounded from above by*

$$\begin{cases} \left(\left(\frac{1+\sqrt{5}}{2}\right)^m + \left(\frac{1-\sqrt{5}}{2}\right)^m\right)\left(\left(\frac{1+\sqrt{5}}{2}\right)^{2m} + \left(\frac{1-\sqrt{5}}{2}\right)^{2m} + 2\right)^{\frac{k+1}{2}} & \text{if } m \text{ is odd;} \\ \left(\left(\frac{1+\sqrt{5}}{2}\right)^m + \left(\frac{1-\sqrt{5}}{2}\right)^m + 2\right)\left(\left(\frac{1+\sqrt{5}}{2}\right)^{2m} + \left(\frac{1-\sqrt{5}}{2}\right)^{2m} + 2\right)^{\frac{k+1}{2}} & \text{if } m \text{ is even.} \end{cases}$$

Theorem 4. $\Phi(F(3,k)) = 3^{k+2} + 1$.

Theorem 5.

$$\Phi(F(4,k)) = 2(2+\sqrt{2})^{k+1} + 2(2-\sqrt{2})^{k+1} + 2^{k+3} + 1.$$

Theorem 6.

$$\Phi(F(5,k)) = 5^{k+2} + 5\left[\left(\frac{5+\sqrt{5}}{2}\right)^k + \left(\frac{5-\sqrt{5}}{2}\right)^k\right] + 1.$$

References:

[1] A. Behmaram, T. Doslic, S. Friedlsnd, Matchings in generalized Fullerene, *Ars Mathematica Contemporanea* **11 (2)** (2016) 301–311.

[2] A. Behmaram, S. Friedland, Upper bounds for perfect matchings in Pfaffian and planar graphs, *Electronic J. Combin.* **20** (2013) #P64, 1–16.

[3] L. Esperet, F. Kardos, A. King, D. Kral and S. Norine, Exponentially many perfect matchings in cubic graphs, *Advances in Mathematics* **227** (2011) 1646–1664.

[4] F. Kardoš, A computer-assisted proof of a Barnette's conjecture: Not only fullerene graphs are Hamiltonian, arXiv:1409.2440v1.

[5] H. W. Kroto, J. R. Heath, S. C. O'Brien, R. F. Curl, R. E. Smaley, C60: Buckminsterfullerene, *Nature* **318** (1985) 162–163.

Minimum Spanning Tree of the Inner Dualist of Honeycomb Graphs

Friday
11.20-11.40

Faqir M. Bhatti

Riphah International University, Pakistan

fmbhatti@riphah.edu.pk

(*joint work with* Khawaja M. Fahd)

The family of honeycomb graphs is a well-known class of graphs. Its inner dual is also an active area of research. The classical graph representations of such graphs do not incorporate the angle at which any of the edges are present. There have been several approaches to keep track of the rotations of the graph. For this purpose we use the He-matrix method that assigns weights to the edges of the graph so that the angle between the vertices and rotations can be preserved.

In the He-Matrix representation there can be six possible orientations of a graph, after reflections and rotations through fixed orientations. In the He-Matrix, edges parallel to the x-axis are given a weight of 1, edges having an angle of 60 degrees are given a weight of 2, and edges with an angle of 120 degrees are assigned a weight of 3.

In this talk we discuss how we partition the edges into different classes according to specific angles $0, 60°, 120°$. Subsets from these classes of edges give the minimum spanning tree. We present the results about cardinality of these subsets. Moreover we derive a linear time algorithm for finding the orientation that gives the least of all minimum spanning trees among all orientations.

An Equivalence Class Decomposition of Finite Metric Spaces via Gromov Products

Tuesday
12.00-12.20

Ayşe Hümeyra Bilge

Kadir Has University, Turkey

ayse.bilge@khas.edu.tr

(*joint work with* Şahin Koçak, Derya Çelik and Metehan İncegül)

Let (X, d) be a finite metric space with elements $P_i, i = 1, \ldots, n$ and with the distance functions d_{ij}. The Gromov Product of the "triangle" (P_i, P_j, P_k) with vertices P_i, P_j and P_k at the vertex P_i is defined by $\Delta_{ijk} = 1/2(d_{ij} + d_{ik} - d_{jk})$ [3]. We show that the collection of Gromov products determines the metric. We call a metric space Δ-generic, if the set of all Gromov products at a fixed vertex P_i has a unique smallest element (for $i = 1, \ldots, n$). We consider the function assigning to each vertex P_i the edge $\{P_j, P_k\}$ of the triangle (P_i, P_j, P_k) realizing the minimal Gromov product at P_i and we call this function the Gromov product structure of the metric space (X, d). We say that two $\Delta-$generic metric spaces (X, d) and (X, d') are Gromov product equivalent, if the corresponding Gromov product structures are the same up to a permutation of X. For $n = 3, 4$ there is one ($\Delta-$generic) Gromov equivalence class and for $n = 5$ there are three ($\Delta-$generic) Gromov equivalence classes [1]. For $n = 6$ we show by computer that there are 26 distinct ($\Delta-$generic) Gromov equivalence classes [2].

References:

[1] J. Koolen, A. Lesser, V. Moulton, Optimal Realizations of Generic Five Point Metrics, *European Journal of Combinatorics* (2009) **30** 1164–1171.

[2] B. Sturmfels, J. Yu, Classification of Six-Point Metrics, *The Electronic Journal of Combinatorics* (2004) **11** R44.

[3] J. Väisälä, Gromov hyperbolic spaces, *Expositiones Mathematicae* (2004) **23** 187–231.

Tuesday
17.40-18.00

Simple Pattern Minimality Problem: two variants and heuristic approaches

Maurizio Boccia

Università del Sannio, Italy

maurizio.boccia@unisannio.it

(*joint work with* Claudio Sterle and Antonio Sforza)

Logical Analysis of Data [2] deals with the classification of huge data sets of binary strings. By using a ternary string (over $\{0, 1, -\}$) called *pattern*, it is possible to identify a group of binary strings *covered by* the pattern. We say that a pattern $\mathbf{p}$ covers a binary string $\mathbf{b}$ if $b_k = p_k$ for each k such that $p_k \in \{0, 1\}$.

A *data set* can be explained by alternative sets of patterns, and many computational problems arise related to the choice of a particular set of patterns for a given instance. The *Simple Pattern Minimality Problem* [3] consists in determining the minimum number of patterns covering exactly the input data set.

In the standard version of the problem no restriction on the generation of patterns is imposed, each string can be covered by more than one pattern. This problem is exactly the *Minimum Disjunctive Normal Form* [1] for which several set covering heuristic approaches are proposed.

A *partitioning* version of the problem where each string of the input data set can be covered by exactly one pattern is defined.

The two problems are solved by using two effective and fast heuristics, tested on large size instances of the SeattleSNPs database [4]. For the covering version of the problem, the proposed heuristic outperforms the existing commercial and freeware logic tools.

References:

[1] M. R. Garey and D. S. Johnson, *Computers and intractability. A guide to the theory of NP completeness. A Series of Books in the mathematical Sciences*, W. H. Freeman and Company, San Francisco (1979).

[2] P. L. Hammer, Partially defined boolean functions and cause-effect relationships, in: *Lecture at the International Conference on Multi-Attrubute Decision Making Via OR-Based Expert Systems.* University of Passau, Passau, Germany (1986).

[3] G. Lancia and P. Serafini, The Complexity of Some Pattern Problems in the Logical Analysis of Large Genomic Data Sets. IWBBIO 2016. *Lecture Notes in Computer Science* **9656**, Springer, Cham.

[4] SeattleSNPs Education Program. 2006. SeattleSNPs. Retrieved March 2008, http://pga.gs.washington.edu.

Non-existence of indecomposable 1-factorizations of the complete multigraph λK_{2n}

Thursday
12.00-12.20

Simona Bonvicini

University of Modena and Reggio Emilia, Italy

simona.bonvicini@unimore.it

(*joint work with* Gloria Rinaldi)

A 1-factorization of the complete multigraph λK_{2n} is said to be indecomposable if it cannot be represented as the union of 1-factorizations of $\lambda_0 K_{2n}$ and $(\lambda - \lambda_0)K_{2n}$, where $\lambda_0 < \lambda$. It is said to be simple if the 1-factors are pairwise distinct. An indecomposable 1-factorization might be simple or not. If $\mathcal{F}$ is an indecomposable 1-factorization of λK_{2n}, then $\lambda < [n(2n-1)]^{n(2n-1)}\binom{2n^3+n^2-n+1}{2n^2-n}$. If $\mathcal{F}$ is simple, then $\lambda < 1 \cdot 3 \cdots (2n-3)$. The existence of an indecomposable 1-factorization of λK_{2n}, simple or not, for every admissible value of λ is an open problem. Partial results about the existence of indecomposable 1-factorizations are known for some values of $2n$ and λ. Non-existence results are known for $2n = 4, 6$. In [2] it is proved that for every $\lambda > 1$ there is no indecomposable 1-factorization of λK_4. It is also proved that there is no indecomposable 1-factorization of $3K_6$. In [3] the authors show that there are precisely three non-isomorphic 1-factorizations of $2K_6$, of which exactly one is indecomposable. The non-existence of indecomposable 1-factorizations of λK_6 for every $\lambda > 3$ is established in [1].

For the next value of $2n$, that is $2n = 8$, only a few results are known. The existence of indecomposable 1-factorizations of λK_8 is proved for every $\lambda \leq 4$ and for $\lambda = 6, 12$

(see [1]). For $\lambda = 6, 12$ the known examples are not simple, for $\lambda \leq 4$ they are simple. Unfortunately, for $2n = 8$ and $\lambda > 4$, no recursive construction that is known in literature can be applied. In [1] it is conjectured that it is not possible to find an indecomposable 1-factorization of λK_8 for every admissible value of λ, that is, for every $\lambda < 28^{28} \cdot \binom{141}{28}$. We give some existence and non-existence results for the case $2n = 8$.

References:

[1] A.H. Baartmans, and W.D. Wallis, Indecomposable factorizations of multigraphs, *Discrete Math.* **78** (1989) 37–43.

[2] C.J. Colbourn, M.J. Colbourn and A. Rosa, Indecomposable 1-factorizations of the complete multigraph, *J. Austral. Math. Soc.* Ser. A **39** (1985) 334–343.

[3] W.D. Wallis, A tournament problem, *J. Austral. Math. Sot.* **248** (1983) 289-291.

Thursday
11.40-12.00

Bipartite graphs in the SSP model of ballistic conduction

Martha Borg

University of Sheffield, UK

mborg1@sheffield.ac.uk

(*joint work with* Patrick W. Fowler, Irene Sciriha)

Bipartite graphs are characterized within the SSP (source-and-sink-potential) model for conduction behaviour. The electron behaviour through the graph is predicted by a generalized eigenvalue equation. The transmission through the graph, to which two semi-infinite wires are attached, can be expressed as a function of the incoming electron energy in terms of four characteristic polynomials, those of the graph and the graphs formed by the deletion of one or both vertices in contact with the external wires. We show that many results depend on counting the zero roots of the structural polynomials. It turns out that what separates the world of omni-conduction and omni-insulation, independent of the connection vertices, is the nullity. Bipartite graphs are neither omni-conductors nor omni-insulators but may be defined as near omni-conductors and omni-insulators. This is due to the restrictions imposed by the spectral properties of bipartite graphs. The classification adopted in terms of a three-letter acronym in combination with the nullity of the graph concentrates on the behavior within and between partite sets, giving rise to 81 classes of possible molecular bipartite devices. After testing all bipartite graphs on up to ten vertices, only 13 were found to be realizable. The nature of bipartite graphs enables the scheme to be classified more finely than for other graphs.

Cross-intersecting families

Monday
15.40-16.00

Peter Borg

University of Malta, Malta

peter.borg@um.edu.mt

Extremal set theory is the study of how small or how large a system of sets can be under certain conditions. The Erdős–Ko-Rado Theorem [4] is a classical result in this field. A variant of the Erdős–Ko-Rado problem is that of determining the maximum sum or the maximum product of sizes of k *cross-t-intersecting* subfamilies $\mathcal{A}_1, \mathcal{A}_2, \dots, \mathcal{A}_k$ of a given family $\mathcal{F}$ of sets, where by 'cross-t-intersecting' we mean that, for every i and j in $\{1, 2, \dots, k\}$ with $i \neq j$, each set in $\mathcal{A}_i$ intersects each set in $\mathcal{A}_j$ in at least t elements. This natural problem has recently attracted much attention. Solutions have been obtained for various important families (as outlined in [1]), such as power sets, levels of power sets, hereditary families, families of permutations, and families of integer sequences. The talk will provide an outline of some of these results. It will focus mostly on the product problem for the family of subsets of $\{1, 2, \dots, n\}$ that have at most r elements. This problem is solved for $t = 1$ in [2]. The problem for the more general setting of *weighted* sets is addressed for any t in [3]; the paper's main result and its applications will be discussed.

References:

[1] P. Borg, The maximum sum and the maximum product of sizes of cross-intersecting families, *European J. Combin.* **35** (2014) 117–130.

[2] P. Borg, A cross-intersection theorem for subsets of a set, *Bull. London. Math. Soc.* **47** (2015) 248–256.

[3] P. Borg, The maximum product of weights of cross-intersecting families, *J. London Math. Soc.* **94** (2016) 993–1018.

[4] P. Erdős, C. Ko and R. Rado, Intersection theorems for systems of finite sets, *Quart. J. Math. Oxford (2)* **12** (1961) 313–320.

On the Super Connectivity of Kneser graphs and Johnson graphs

Monday
16.00-16.20

Gülnaz Boruzanlı Ekinci

Ege University, Turkey

gulnaz.boruzanli@ege.edu.tr

(*joint work with* John Baptist Gauci)

A vertex cut of a connected graph G is a set of vertices whose deletion disconnects G. A connected graph G is super–connected if the deletion of every minimum vertex cut of G isolates a vertex. The super–connectivity is the size of the smallest vertex cut of G such that each resultant component does not have an isolated vertex. The Kneser graph $KG(n,k)$ is the graph whose vertices are the k-subsets of $\{1,2,...,n\}$ and two vertices are adjacent if the k-subsets are disjoint. The Johnson graph $J(n,k)$ is the graph whose vertices are also the k-subsets of $\{1,2,...,n\}$, but two vertices are adjacent if the corresponding k-subsets have exactly $k-1$ elements in common. We use Baranyai's Theorem on the decompositions of complete hypergraphs to show that the Kneser graphs $KG(n,2)$ are super–connected when $n \geq 5$ and that their super–connectivity is $\binom{n}{2} - 6$. We also show that the super–connectivity of $J(n,2)$ is $3(n-3)$ when $n \geq 6$. These results are of interest especially in the study of reliability and fault tolerance of interconnection networks, since these graph families are good candidates for such networks.

On χ-binding functions for the classes of P_5-, $2K_2$-, and $K_{1,3}$-free graphs

Tuesday
17.40-18.00

Christoph Brause

TU Bergakademie Freiberg, Germany

brause@math.tu-freiberg.de

A graph G is called *k-colourable* if its vertices can be coloured with k colours so that adjacent vertices receive distinct colours. The smallest integer k such that a given graph G is k-colourable is called the *chromatic number* of G, denoted by $\chi(G)$. It is well-known that $\omega(G) \leq \chi(G) \leq \Delta(G) + 1$ holds for any graph G, where $\omega(G)$ denotes the clique number and $\Delta(G)$ the maximum degree of G.

By an old result of Erdős [4], there does not exist a χ-binding function f, which is a function $f : \mathbb{N} \to \mathbb{R}_0^+$ such that $\chi(G) \leq f(\omega(G))$ for any graph G. However, Gyárfás asks in [5], whether or not there exists a χ-binding function for the class of graphs having no induced subgraph isomorphic to some given graph F. It is known by Erdős' result that if there exists such a function for the class of F-free graphs, then F must be a forrest. Unfortunately, only for a few forrests F, a χ-binding function for the class of F-free graphs is known and, for most of them, the right order of magnitute is unknown.

In this talk, we will present χ-binding functions for the classes of P_5-free graphs, of $2K_2$-free graphs, and of $K_{1,3}$-free graphs. Furthermore, we will study the chromatic number in subclasses defined in terms of a second forbidden subgraph. By analysing these results, we will obtain lower bounds for χ-binding functions for our initial classes defined by one forbidden subgraph.

This talk bases on results in [1,2,3].

References:

[1] C. Brause, T.D. Doan, and I. Schiermeyer, On the chromatic number of $(P_5, K_{2,t})$-free graphs, *Electronic Notes in Discrete Mathematics* **55** (2016) 121–130.

[2] C. Brause, P. Holub, A. Kabela, Z. Ryjáček, I. Schiermeyer, and P. Vrána, On forbidden subgraphs for $K_{1,3}$-free perfect graphs, manuscript.

[3] C. Brause, B. Randerath, I. Schiermeyer, and E. Vumar, On the chromatic number of $2K_2$-free graphs, manuscript.

[4] P. Erdős, Graph theory and probability, *Canad. J. Math.* **11** (1959), 34–38.

[5] A. Gyárfás, Problems from the world surrounding perfect graphs, In Proc. Int. Conf. on Comb. Analysis and Applications (Pokrzywna, 1985), *Zastos. Mat.* **19** (1987) 413–441.

Clique coverings and claw-free graphs

Monday
15.20-15.40

Csilla Bujtás

University of Pannonia, Hungary

bujtas@dcs.uni-pannon.hu

(*joint work with* Akbar Davoodi, Ervin Győri and Zsolt Tuza)

Let $\mathcal{C}$ be a clique covering for $E(G)$ and let v be a vertex of G. The valency of vertex v (with respect to $\mathcal{C}$), denoted by $val_{\mathcal{C}}(v)$, is the number of cliques in $\mathcal{C}$ containing v. The local clique cover number of G, denoted by $\mathrm{lcc}(G)$, is defined as the smallest integer k, for which there exists a clique covering for $E(G)$ such that $val_{\mathcal{C}}(v)$ is at most k, for every vertex $v \in V(G)$. This parameter may be interpreted as a variety of different invariants of the graph. For example, $\mathrm{lcc}(G)$ is the minimum integer k for which G is the line graph of a k-uniform hypergraph.

We consider the following two conjectures:

(1) For every graph G of order n, $\mathrm{lcc}(G)+\mathrm{lcc}(\overline{G}) \le n$ holds.
(Proposed by R. Javadi, Z. Maleki, and B. Omoomi in 2012.)

(2) For every graph G of order n, $\mathrm{lcc}(G) + \chi(G) \leq n + 1$ holds.
(Proposed by A. Davoodi, R. Javadi, and B. Omoomi as a weakening of (1).)

Among other results, we prove that (1) is true if $\alpha(G) = 2$, and (2) holds if G is a claw-free graph.

Wednesday
11.20-11.40

Exploring soft graphs

Jean-Guy Caputo

INSA de Rouen, France

caputo@insa-rouen.fr

(*joint work with* Arnaud Knippel)

In this presentation, we are interested in graphs whose graph Laplacian has an eigenvector with a null component. The graph Laplacian is the matrix of node degrees minus the adjacency matrix; we refer to [3] for definitions and results on graph spectra. In a previous work [1] we called *soft node* a vertex corresponding to such a component. In the case of a multiple eigenvalue, any component of an eigenvector may be zero and we call *absolute soft node* a vertex with value zero for all eigenvectors in the subspace.

Here we call *soft graphs* graphs with a soft node. We present a classification [2] of λ-soft graphs, sorted by value of λ, with all soft graphs with up to 6 nodes, as well as some particular classes of graphs. This shows a structure with some graph mappings, and suggests that we can build soft graphs and find eigenvectors with soft nodes combinatorially.

Acknowledgements. This work is part of project XTerM, funded with the support from the European Union with the European Regional Development Fund (ERDF) and from the Regional Council of Normandie.

References:

[1] J.-G. Caputo, A. Knippel and E. Semo, Oscillations of networks: the role of soft nodes, *Journal of Physics A: Mathematical and Theoretical* **46** (2013) 18pp.

[2] J.-G. Caputo and A. Knippel, Classification of λ-soft graphs, *technical report* (2017).

[3] D. Cvetkovic and P. Rowlinson and S. Simic, *An Introduction to the Theory of Graph Spectra*, London Mathematical Society Student Texts (No. 75), (2001).

Generalized connectivity and strong product graph

Thursday
16.00-16.20

Rocío M. Casablanca

University of Seville, Spain

rociomc@us.es

(*joint work with* Encarnación Abajo, Ana Diánez and Pedro García-Vázquez)

Let G be a connected graph with n vertices and let k be an integer such that $2 \leq k \leq n$. The generalized connectivity $\kappa_k(G)$ of G is the greatest positive integer ℓ for which G contains at least ℓ internally disjoint trees connecting S for any set $S \subseteq V(G)$ of k vertices. It represents a generalization of the concept of vertex connectivity. Clearly, $\kappa_2(G)$ is just the connectivity $\kappa(G)$, which is the reason why one addresses $\kappa_k(G)$ as the generalized connectivity of G. It measures the capability to connect any set of k vertices in a network.

Products of graphs provide important methods to construct bigger graphs and play a key role in design and analysis of networks. Our purpose is to study the 3-generalized connectivity of the strong product of graphs. The *strong product* $G_1 \boxtimes G_2$ of two connected graphs G_1 and G_2 is the graph with vertex set $V(G_1) \times V(G_2)$ in which two vertices (x_1, x_2) and (y_1, y_2) are adjacent if $x_1 = y_1$ and $x_2y_2 \in E(G_2)$, or $x_1y_1 \in E(G_1)$ and $x_2 = y_2$, or $x_1y_1 \in E(G_1)$ and $x_2y_2 \in E(G_2)$.

We focus on the case when $k = 3$. We study the generalized 3-connectivity for the strong product $G_1 \boxtimes G_2$ of two connected graphs G_1 and G_2 with at least three vertices and girth at least five, and we prove the sharp bound

$$\kappa_3(G_1 \boxtimes G_2) \geq \kappa_3(G_1)\kappa_3(G_2) + \kappa_3(G_1) + \kappa_3(G_2) - 1.$$

References:

[1] E. Abajo, R.M. Casablanca, A. Diánez, P. García-Vázquez, On average connectivity of the strong product of graphs, *Discrete Appl. Math.* **161** (2013) 2795–2801.

[2] E. H. Li, X. Li and Y. Sun, The generalized 3-connectivity of Cartesian product graphs, *Discrete Mathematics and Theoretical Computer Science* **14** (2012) 43–54.

[3] K. Menger, Zur allgemeinen Kurventheorie, *Fund. Math.* **10** (1927) 96–115.

The Clustered and Bottleneck Clustered Selected-Internal Steiner Tree Problems

Yen Hung Chen

University of Taipei, Taiwan

yhchen@utaipei.edu.tw

In the paper, we study two variants of the clustered Steiner tree problem [4], namely the clustered selected-internal Steiner tree problem and the bottleneck clustered selected-internal Steiner tree problem, respectively. Given a complete graph $G = (V, E)$, with nonnegative edge costs, two subsets $R \subset V$ and $R' \subset R$, a partition $\mathcal{R} = \{R_1, R_2, \ldots, R_k\}$ of R, $R_i \cap R_j = \phi$, $i \neq j$ and $\mathcal{R}' = \{R'_1, R'_2, \ldots, R'_k\}$ of R', $R'_i \subset R_i$, a clustered Steiner tree is a tree T of G spanning all vertices in R such that T can be cut into k subtrees T_i by removing $k-1$ edges and each subtree T_i spans all vertices in R_i, $1 \leq i \leq k$. The total cost of a tree is defined to be the sum of the costs of all its edges. A clustered selected-internal Steiner tree of G is a clustered Steiner tree for R if all vertices in R'_i are internal vertices of T_i, $1 \leq i \leq k$. The clustered selected-internal Steiner tree problem (respectively, the bottleneck clustered selected-internal Steiner tree problem) is concerned with finding a clustered selected-internal Steiner tree T for R in G whose total cost (respectively, the cost of the largest edge) of T is minimized. The clustered selected-internal Steiner tree problem and the bottleneck clustered selected-internal Steiner tree problem are NP-hard, since the selected-internal Steiner tree and the bottleneck selected-internal Steiner tree problems [2,3] are their special versions when $k = 1$, respectively. Applications of two problems include the multicast routing and the facility location in telecommunications and wavelength-division multiplexing (WDM) optical networks. In this paper, we present the first known approximation algorithms with performance ratio $\rho + 4$ and 4 for the clustered selected-internal Steiner tree problem and the bottleneck clustered selected-internal Steiner tree problem if the cost function is metric (i.e., the costs of edges satisfy the triangle inequality), respectively, where ρ is the best-known performance ratio for the Steiner tree problem (currently $\rho = \ln 4 + \epsilon \approx 1.39$ [1]).

References:

[1] J. Byrka, F. Grandoni, T. Rothvos and L. Sanita, Steiner Tree Approximation via Iterative Randomized Rounding, *Journal of the ACM* **60** (2013) 6.

[2] Y.H. Chen, The bottleneck selected-internal and partial terminal Steiner tree problems, *Networks* **68** (2016) 331–339.

[3] S.Y. Hsieh and S.C. Yang, Approximating the selected-internal Steiner tree, *Theoretical Computer Science* **381** (2007) 288–291.

[4] B.Y. Wu and C.W. Lee, On the clustered Steiner tree problem, *Journal of Combinatorial Optimization* **30** (2015) 370–386.

Skolem Labellings of Graphs with Large Chordless Cycles

Monday
16.40-17.00

Nancy E. Clarke

Acadia University, Canada

nancy.clarke@acadiau.ca

(*joint work with* Asiyeh Sanaei)

In this talk, we consider Skolem (vertex) labelling and present (hooked) Skolem labellings for generalized Dutch windmills whenever such labellings exist. Specifically, we show that generalized Dutch windmills with more than two cycles cannot be Skolem labelled and that those composed of two cycles of lengths m and n, $n \geq m$, cannot be Skolem labelled if and only if $n - m \equiv 3, 5 \pmod 8$ and m is odd. Showing that a Skolem labelling does not exist is, in general, a complex problem and we present a novel technique for doing so.

Independent Sets of Families of Graphs via Finite State Automata

Friday
11.40-12.00

Pietro Codara

Artificial Intelligence Research Institute (IIIA-CSIC), Spain

codara@di.unimi.it

(*joint work with* Ottavio M. D'Antona, Massimo Galasi and Giovanna Lavado)

An *independent set* of a graph is a set of pairwise non-adjacent vertices of the graph. The main goal of this work is the enumeration of the independent sets of a wide set of families of graphs that we call *telescopic families of graphs*, TFGs. What is particularly interesting in our approach is that we obtain our results via finite state automata. Given a TFG, $\{G_n\}_{n \geq 0}$ say, we show how to build its *independence automaton*, that is a deterministic finite automaton that accepts a language in which the number of n-symbol words equals the number of independent sets of G_n, for any $n \geq 0$. Our work has been inspired by the paper [1] that deals, among other things, with the enumeration of the independent sets of grid graphs, i.e. Cartesian products of paths. Needless to say, grid graphs make a TFG.

Acknowledgements. Pietro Codara is supported by an INdAM-COFUND-2012 Marie Curie fellowship (project "LaVague"). Giovanna Lavado is supported by Dipartimento di Informatica, Università degli Studi di Milano.

References:

[1] N. J. Calkin and H. S. Wilf, The number of independent sets in a grid graph, *SIAM Journal on Discrete Mathematics* **11-1** (1998) 54–60.

Tuesday
17.20-17.40

The Forgotten Topological Index of some Carbon Base Nanomaterials

Özge Çolakoğlu Havare

Mersin University, Turkey

ozgecolakoglu@mersin.edu.tr

(*joint work with* Ali Kemal Havare)

Molecular topological indices are numerical descriptors of molecular structure obtained via a molecular graph G [1]. Furthermore, computing the connectivity indices of molecular graphs is a meaningful branch in chemical graph theory. It is of interest to find topological indices which correlate well with chemical properties of the chemical molecules. Lately, the most common index is the forgotten topological index, which has been defined following the first and second Zagreb topological indices. The F-index or the forgotten topological index $F(G)$ is a vertex degree based topological index and it can be expressed as,

$$F(G) = \sum_{v \in V(G)} d(v)^3 = \sum_{uv \in E(G)} d(u)^2 + d(v)^2$$

where $d(u)$ denotes the degree of u [2,4]. Recently, the forgotten index or the F-index was shown to have an exceptional applicative potential. In addition, there are many topological descriptors that are applicable in QSPR/QSAR. In this study, the F-index of $TUSC_5C_7$, $TUHAC_5C_7$ and $TUHAC_5C_6C_7$ nanotubes which is a family of nanostructures are computed [3]. This analytic approach is given to compute the F-index of linear phenylenes and cyclic phenylenes . The computing is correlated with the chemical properties of nanostructures and it gives information about their physical features.

References:

[1] G. Cash, S. Klavzar, M. Petkovsek, Three Methods for Calculation of the Hyper-Wiener Index of Molecular Graphs, *J. Chem. Inf. Comput. Sci* **42** (2002) 571–576.

[2] N. De, Sk. Md. A. Nayeem, Computing the F-index of Nanostar Dendrimers, *Pacific Science Review A: Natural Science and Engineering* **18** (2016) 14–21.

[3] M. R. Farahani, The first and second Zagreb Indices, First and Second Zagreb Polynomials of $HAC_5C_6C_7$[p,q] and HAC_5C_7[p,q] Nanotubes, *Int. J. Nanosci. Nanotechnol.* **8** (2012) 175–180

[4] I. Gutman and B. Furtula, A forgotten topological index, *J. Math. Chem.* **53** (2015) 1184–1190.

A construction of regular Hadamard matrices

Monday
12.20-12.40

Dean Crnković

University of Rijeka, Croatia

deanc@math.uniri.hr

(*joint work with* Ronan Egan)

A Hadamard matrix of order m is a $(m \times m)$ matrix $H = (h_{i,j})$, $h_{i,j} \in \{-1, 1\}$, satisfying $HH^T = H^T H = mI_m$, where I_m is an $(m \times m)$ identity matrix. A Hadamard matrix is called regular if the row and column sums are constant. The existence of a regular Hadamard matrix is well known to be equivalent to the existence of a symmetric $(4m^2, 2m^2 - m, m^2 - m)$ design, also known as a Menon design. It is conjectured that a regular Hadamard matrix of order $4m^2$ exists for every positive integer m. In this talk we give a method of constructing regular Hadamard matrices using conference graphs and Hadamard designs with skew incidence matrices.

The number of P-vertices in a matrix with maximum nullity

Monday
16.40-17.00

Henrique F. da Cruz

Universidade da Beira Interior, Portugal.

hcruz@ubi.pt

(*joint work with* Rosário Fernandes)

Let T be a tree with $n \geq 2$ vertices. Let $\mathcal{S}(T)$ be the set of all real symmetric matrices whose graph is T. Let $A \in \mathcal{S}(T)$, and $i \in \{1, \ldots, n\}$. We denote by $A(i)$ the principal submatrix of A obtained after deleting the row and column i. We set $m_A(0)$ for the multiplicity of the eigenvalue zero in A (the nullity of A). When $m_{A(i)}(0) = m_A(0) + 1$, we say that i is a P-vertex of A. As usual, $M(T)$ denotes the maximum nullity occurring in matrices of $\mathcal{S}(T)$. We determine an upper bound and a lower bound for the number of P-vertices in a matrix $A \in \mathcal{S}(T)$ with nullity $M(T)$, and we prove that if z is an integer between these two bounds, then there is a matrix $A \in \mathcal{S}(T)$ with maximum nullity, and with z P-vertices.

Acknowledgements: Henrique F. da Cruz was partially supported for this work by national funds of FCT- Foundation for Science and Technology under the project UID/MAT/00212/2013.

Monday
11.40-12.00

On the Mathon bound for regular near hexagons

Bart De Bruyn

Ghent University, Belgium

Bart.DeBruyn@UGent.be

A finite graph Γ of diameter $d \geq 2$ is called *distance-regular* if there exist constants a_i, b_i, c_i ($i \in \{0, 1, \ldots, d\}$) such that $|\Gamma_1(x) \cap \Gamma_i(y)| = a_i$, $|\Gamma_1(x) \cap \Gamma_{i+1}(y)| = b_i$ and $|\Gamma_1(x) \cap \Gamma_{i-1}(y)| = c_i$ for any two vertices x and y at distance i from each other. A finite graph Γ is called a *regular near hexagon* with parameters (s, t, t_2) if it is a distance-regular graph with diameter 3, intersection array $\{b_0, b_1, b_2; c_1, c_2, c_3\} = \{s(t+1), st, s(t-t_2); 1, t_2+1, t+1\}$ and does not contain any $K_{1,1,2}$'s as induced subgraphs (i.e. no complete graphs on four vertices minus an edge). If $s \neq 1$, then an inequality due to Rudi Mathon states that $t \leq s^3 + t_2(s^2 - s + 1)$. In fact, in the special case that $t = s^3 + t_2(s^2 - s + 1)$, it can be shown that any distance-regular graph of diameter 3 and intersection array $\{b_0, b_1, b_2; c_1, c_2, c_3\} = \{s(t+1), st, s(t-t_2); 1, t_2+1, t+1\}$ cannot contain $K_{1,1,2}$'s as induced subgraphs and hence must be a regular near hexagon.

In my talk, I will discuss some new proofs of this Mathon inequality. These proofs give additional structural information about the regular near hexagon in case t attains the Mathon bound. This additional structural information can be (and has already been) useful for showing the non-existence of distance-regular graphs with certain parameters. Part of this work is joint with Frédéric Vanhove.

References:

[1] B. De Bruyn and F. Vanhove. Inequalities for regular near polygons, with applications to m-ovoids. *European J. Combin.* **34** (2013) 522–538.

[2] B. De Bruyn and F. Vanhove. On Q-polynomial regular near $2d$-gons. *Combinatorica* **35** (2015) 181–208.

[3] B. De Bruyn. A combinatorial characterization of extremal generalized hexagons. Preprint.

[4] B. De Bruyn. An alternative proof of the Mathon bound for regular near hexagons. Preprint.

The Hamilton Cycle Problem for locally traceable and locally Hamiltonian graphs

Tuesday
11.40-12.00

Johan de Wet

University of South Africa, South Africa

jpdewet314@gmail.com

(*joint work with* Marietjie Frick and Susan van Aardt)

We say a graph G is locally $\mathcal{P}$ if for each vertex v in G the open neighbourhood of v induces a graph with property $\mathcal{P}$. The Hamilton Cycle Problem (HCP) is the problem of deciding whether a graph contains a Hamilton cycle. Gordon et al. [1] showed that the HCP for locally connected graphs is NP-complete for graphs with maximum degree 7, and they conjectured that 7 is the smallest value of the maximum degree for which this is true. However, we show that the HCP is NP-complete for locally traceable (LT) graphs with maximum degree 6. If $\mathcal{R}$ is a set of nonnegative integers, we say a graph G is $\mathcal{R}$-regular if the degrees of all the vertices in $V(G)$ are elements of $\mathcal{R}$. We show that the HCP is NP-complete for $\{2,6\}$-, $\{3,6\}$-, $\{5,6\}$-, and r-regular LT graphs, where $r \geq 6$.

Maximal planar graphs are locally hamiltonian (LH), and it is known that the HCP for maximal planar graphs is NP-complete [2], but no attention has thus far been given to the smallest value of the maximum degree of the graph for which this is true. We show that the HCP is NP-complete for LH graphs with maximum degree 9 and for LH graphs that are $\{3,9\}$-, $\{3,10\}$, and r-regular, for $r \geq 11$. Finally, we show that the HCP for k-connected LH graphs is NP-complete for every $k \geq 3$.

References:

[1] V.S. Gordon, Y.L. Orlovich, C.N. Potts and V.A. Strusevich, Hamiltonian properties of locally connected graphs with bounded vertex degree, *Discrete Applied Math. 159* (2011) 1759–1774.

[2] V. Chvátal, Hamiltonian cycles, in: E.L. Lawler, J.K. Lenstra, A.H.G. Rinnooy Kan, D.B. Shmoys (eds.), *The Travelling Salesman problem: A Guided Tour of Combinatorial Optimization*, Wiley-Intersci. Ser. Discrete Math., Wiley, Chichester, 1985, 403–429.

Monday
12.00-12.20

Recent progress on partial difference sets in Abelian groups

Stefaan De Winter

Michigan Technological University, U.S.A.

sgdewint@mtu.edu

(*joint work with* Zeying Wang)

Partial difference sets were introduced by Bose and Cameron, and a detailed study was initiated by S.L. Ma in the 80's. It is well known that these special subsets of a finite group are equivalent to strongly regular Cayley graphs. The classical tools to study partial difference sets are the use of characters and computations in the group ring. Recently, jointly with Z. Wang, we have introduced an approach to partial difference sets in Abelian groups based on linear algebra. This has allowed us to obtain some rather strong results. We finalized the classification of parameters for which there exists a strongly regular Cayley graph of valency at most 100 on an Abelian group. This required proving non-existence for 18 parameter sets that had been open for more than 20 years. We obtained a complete classification of partial difference in Abelian groups of order $4p^2$, p an odd prime, and are currently finalizing a similar classification for Abelian groups of order $9p^2$. The importance here lies in the fact that few general classification results for partial difference sets are known. We are also able to obtain some well-known multiplier results and exponent bounds using our approach. In this talk I plan to explain the general ideas underlying our approach, provide an overview of our results on partial difference sets in Abelian groups, and explain how our technique can also be used to study related objects, such as skew-Hadamard difference sets.

Monday
17.00-17.20

New results on distance spectrum of special classes of trees

Renata Del-Vecchio

Universidade Federal Fluminense, Brazil

renata@vm.uff.br

(*joint work with* M. A. de Freitas and J. S. Nascimento)

In this work we study the spectrum of the distance matrix of trees.

In [2], Merris has shown that, in the case of trees, -2 is an eigenvalue of the distance matrix (here called a D-eigenvalue) with “high” multiplicity, presenting a lower bound for the multiplicity of this eigenvalue. In the present work we obtain the exact value of the multiplicity of -2 as a D-eigenvalue for caterpillars, brooms and double brooms. Furthermore, we completely characterize the spectrum of these trees when its diameter is three or four.

A known fact about graphs is that the number of distinct eigenvalues of the adjacency, Laplacian and signless Laplacian matrices is at least the diameter plus one unit. For the distance matrix, meanwhile, it generally does not happen. In [1] a lower bound for the number of distinct eigenvalues of the distance matrix of any tree, as a function of its diameter, is provided. We obtain here, for caterpillars, brooms and double brooms, the exact number of distinct eigenvalues of the distance matrix, as a function of the diameter of the graph.

References:

[1] G. Aalipour, A. Abiad, Z. Berikkyzy, *et al.* On the distance spectra of graphs, *Linear Algebra and its Applications* **497** (2016) 66-87.

[2] R. Merris, The distance spectrum of a tree, *Journal of Graph Theory* **14**, n. 3 (1990) 365-369.

Degree Sequences of Hypergraphs, Self-Complementary and Almost Self-Complementary 3-uniform Hypergraphs

Tuesday
17.00-17.20

Charusheela Deshpande

College of Engineering, SPPU, Pune, India

dcm.maths@coep.ac.in

(*joint work with* Lata Kamble and Bhagyashri Bam)

Characterizing the degree sequences of hypergraphs is a long standing open problem and not much work has been done in this area. We have characterized the degree sequences of linear hypergraphs by giving necessary and sufficient conditions very similar to those given by Erdős and Gallai for graphical sequences.

We have studied factorization of 3-uniform hypergraphs into two isomorphic factors and have constructed all such hypergraphs along with the properties of complementing permutations.

References:

[1] C. Deshpande and B. Bam and N. Bhave, Characterisation of degree sequences of linear hypergraphs, *Journal of Indian Mathematical Society* **Vol 75** (2008) 151-160.

[2] C. Deshpande, L. Kamble, B. Bam, The existence of quasi regular and bi-regular complementary 3-uniform hypergraphs, *Discussiones Mathematicae Graph Theory* **36** (2016) 419-426.

[3] C. Deshpande, L. Kamble, B. Bam, Almost Self-Complementary 3 Uniform Hypergraphs,*Discussiones Mathematicae Graph Theory* **37** (2017) 131-140.

Thursday
16.40-17.00

The Fibonacci numbers of the composition of graphs

Loiret A. Dosal-Trujillo

Universidad Nacional Autonoma de Mexico, Mexico

loiret@matem.unam.mx

(*joint work with* Hortensia Galeana-Sánchez)

A subset S of vertices of a graph G is said to be independent if for every two vertices $u, v \in S$ there is no edge between them. The Fibonacci number of a graph G is the total number of independent vertex sets of G. This concept was introduced by Prodinger and Tichy in [2]. They proved that the number of independent vertex sets of P_n, the path of order n is precisely a Fibonacci number, and for C_n the cycle of order n is the n-Lucas number. The problem to find the Fibonacci number of a graph is an NP-complete problem. In [1] we proved that for every $r \in \mathbb{Z}^+$ and for all $n \geq r+1$, the Fibonacci numbers of $C_{n[r]}$, the circulant graphs of order n with consecutive jumps $(1, 2, \ldots, r)$ and for several subgraphs of this family are characterized by some sequences which generalize the Fibonacci and Lucas sequences.

Given a graph G and a family of graphs $\alpha = (\alpha_v)_{v \in V(G)}$ without vertices in common, the Zykov sum $\sigma(G, \alpha)$ is the graph with vertex set $\cup_{v \in V(G)} V(\alpha_v)$, and edge set $\cup_{v \in V(G)} E(\alpha_v) \cup \{xy : x \in V(\alpha_u), y \in V(\alpha_v) \text{ and } uv \in E(G)\}$. The composition G with H of two disjoint graphs G and H, is the graph $\sigma(G, \alpha)$, where α_v is isomorphic to the graph H, for every $v \in V(G)$. In this talk, we will explain how to obtain the Fibonacci number of the composition of graphs.

References:

[1] L. A. Dosal-Trujillo and H. Galeana-Sánchez, The Fibonacci numbers of certain subgraphs of circulant graphs, *AKCE International Journal of Graphs and Combinatorics* **12** (2015) 92–103.

[2] H. Prodinger and R. F. Tichy, Fibonacci numbers of graphs, *The Fibonacci Quarterly* **20** (1982) 16–21.

2-proper connection of graphs

Monday
16.20-16.40

Trung Duy Doan

TU Bergakademie Freiberg, Germany

trung.doanduy@gmail.com

(*joint work with* Christoph Brause and Ingo Schiermeyer)

A path in an edge-coloured graph is called a *proper path* if every two consecutive edges receive distinct colours. An edge-coloured graph G is called *k-properly connected* if every two vertices are connected by at least k internally pairwise vertex-disjoint proper paths. The *k-proper connection number* of a connected graph G, denoted by $pc_k(G)$, is the smallest number of colours that are needed in order to make G k-properly connected. In this paper, we study the 2-proper connection number $pc_2(G)$ of 2-connected graphs. We prove a new upper bound of $pc_2(G)$, determine $pc_2(G) = 2$ for several classes of 2-connected graphs and $pc_2(G\Box H)$, where $G\Box H$ is the Cartesian product of two nontrivial connected graphs.

References:

[1] E. Andrews, C. Lumduanhom, E. Laforge, and P. Zhang, On Proper-Path colourings in Graphs, *Journal of Combinatorial Mathematics and Combinatorial Computing* **97** (2016) 189–207.

[2]V. Borozan, S. Fujita, A. Gerek, C. Magnant, Y. Manoussakis, L. Montero, and Z. Tuza, Proper connection of graphs, *Discrete Math.* **312(17)** (2012) 2550–2560.

Restricted size Ramsey numbers for some graphs

Monday
17.00-17.20

Tomasz Dzido

University of Gdańsk, Poland

tdz@inf.ug.edu.pl

(*joint work with* Joanna Cyman)

Let F, G, and H be simple graphs. We say $F \to (G, H)$ if for every 2-coloring of the edges of F there exists a monochromatic G or H in F. The Ramsey number $r(G, H)$ is defined as

$$r(G, H) = \min\{|V(F)| : F \to (G, H)\},$$

while the restricted size Ramsey number $r^*(G, H)$ is defined as

$$r^*(G, H) = \min\{|E(F)| : F \to (G, H), |V(F)| = r(G, H)\}.$$

In this talk, we determine previously unknown restricted size Ramsey numbers and give some new bounds for some graphs.

Monday
12.20-12.40

Toughness and treewidth

Mark Ellingham

Vanderbilt University, U.S.A.

mark.ellingham@vanderbilt.edu

(*joint work with* Songling Shan, Dong Ye and Xiaoya Zha)

We discuss two results involving toughness and treewidth. First, Jackson and Wormald conjectured that for $k \geq 2$ every $\frac{1}{k-1}$-tough graph has a spanning closed walk using every vertex at most k times. We show that this is true for graphs of treewidth at most 2, or equivalently for K_4-minor-free graphs. In fact, we prove the stronger result that for $k \geq 2$ every $\frac{1}{k-1}$-tough graph of treewidth at most 2 has a spanning tree of maximum degree at most k. Second, computing toughness is NP-hard for general graphs. We show that toughness, or the truth of certain conditions related to toughness, can be determined in polynomial time for graphs of bounded treewidth.

Thursday
11.20-11.40

Navigating Between Packings of Graphic Sequences

Péter L. Erdős

MTA, A. Rényi Institute of Mathematics, Hungary

erdos.peter@renyi.mta.hu

(*joint work with* Mike Ferrara and Stephen Hartke)

A nonnegative integer sequence π is *graphic* if it is the degree sequence of some graph G. In this case we say that G *realizes* or is a *realization* of π. Graphic sequences $\pi_1 = (d_1^{(1)}, \ldots, d_n^{(1)})$ and $\pi_2 = (d_1^{(2)}, \ldots, d_n^{(2)})$ *pack* if there exist edge disjoint realizations G_1 and G_2 of π_1 and π_2, respectively, on vertex set $\{v_1, \ldots, v_n\}$ such that for $j \in \{1, 2\}$, $d_{G_j}(v_i) = d_i^{(j)}$ for all $i \in \{1, \ldots, n\}$. In this case, we say that (G_1, G_2) is a (π_1, π_2)-*packing*.

A clear necessary condition for graphic sequences π_1 and π_2 to pack is that $\pi_1 + \pi_2$, their componentwise sum, is also graphic. It is known, however, that this condition is not sufficient, and furthermore that the general problem of determining if two sequence pack is NP-complete [1]. In 1973 S. Kundu [2] proved that if π_2 is almost regular, that is each element is from $\{k-1, k\}$, then π_1 and π_2 pack if and only if $\pi_1 + \pi_2$ is graphic. This result was originally conjectured in 1972 by Rao and Rao [3]. For $k = 1$ it was conjectured by B. Grünbaum in 1970.

In this talk we will consider graphic sequences π with the property that $\pi + \mathbf{1}$ is graphic. By Kundu, the sequences π and $\mathbf{1}$ pack, and there exist edge disjoint realizations G and $\mathcal{I}$ where $\mathcal{I}$ is a 1-factor. We call such a $(\pi, \mathbf{1})$ packing a *Kundu realization* of $\pi + \mathbf{1}$.

Given realizations G and G' of a graphic sequence π, a classical result of Petersen (which was rediscovered independently several times) states that it is possible to transform G into G' through a sequence of *swap operations* (also known as *switch* or *rewiring* operations), wherein the edges and non-edges of an alternating 4-cycle are interchanged.

Assume that π is a graphic sequence in which each term is at most $c\sqrt{n}$ that packs with $\mathbf{1}$. Furthermore let $\mathcal{J}$ be a given, particular 1-factor. Our main result is that one can find realization $G_{\mathcal{J}}$ of π such that $(G_{\mathcal{J}}, \mathcal{J})$ is a $(\pi, \mathbf{1})$-packing.

As a byproduct we also solve the following problem: Let $(G, \mathcal{I})$ and $(G', \mathcal{J})$ be two Kundu realizations of $\pi + \mathbf{1}$. We show that it is possible to transform $(G, \mathcal{I})$ into $(G'\mathcal{J})$ via a sequence of swap operations that naturally generalize the classical notion of an alternating 4-cycle to the setting of graphic sequence packing. Each intermediate realization of $\pi + \mathbf{1}$ is also a Kundu realization.

Acknowledgements: Péter L. Erdős was supported in part by the National Research, Development and Innovation – NKFIH grant K 116769and SNN 116095.

References:

[1] C. Dürr, F. Guiñez and C. Matamala, Reconstructing 3-colored grids from horizontal and vertical projections is NP-hard: a solution to the 2-atom problem in discrete tomography, *SIAM J. Discrete Math.*, **26** (2012) 330–352.

[2] S. Kundu, The k-factor conjecture is true, *Discr. Math.* **6** (1973) 367–376.

[3] A.R. Rao and S.B. Rao, On factorable degree sequences, *J. Comb. Theory B* **13** (1972) 185–191.

Autotopism stabilized colouring games on rook's graphs

Thursday
16.40-17.00

Raúl M. Falcón

University of Seville, Spain

rafalgan@us.es

(*joint work with* Stephan Dominique Andres)

Based on the fact that every partial colouring of the rook's graph $K_r \Box K_s$ is uniquely related to an $r \times s$ partial Latin rectangle, this work deals with the Θ-*stabilized colouring game* on the graph $K_r \Box K_s$. This is a variant of the classical colouring game on finite graphs [1,2,6,7] so that each move must respect a given autotopism Θ of the resulting partial Latin rectangle. The complexity of this variant is examined by means of its Θ-*stabilized game chromatic number*, which depends in turn on the cycle structure of the autotopism under consideration. Based on the known classification of such cycle structures [3,4,5,8], we determine in a constructive way the game chromatic number associated to those rook's graphs $K_r \square K_s$, for which $r \leq s \leq 8$.

References:

[1] H. L. Bodlaender, On the complexity of some coloring games, *Int. J. Found. Comput. Sci.* **2** (1991) 133–147.

[2] S. D. Andres and R. M. Falcón, Colouring games based on autotopisms of Latin hyper-rectangles, preprint.

[3] R. M. Falcón, Cycle structures of autotopisms of the Latin squares of order up to 11, *Ars Comb.* **103** (2012) 239–256.

[4] R. M. Falcón, The set of autotopisms of partial Latin squares, *Discrete Math.* **313** (2013) 1150–1161.

[5] R. M. Falcón and R. J. Stones, Classifying partial Latin rectangles, *Electron. Notes Discrete Math.* **49** (2015), 765–771.

[6] M. Gardner, *Mathematical games*, Scientific American (April, 1981), 23.

[7] M. Schlund, Graph decompositions, Latin squares, and games. Diploma Thesis, TU München, 2011.

[8] D. S. Stones, P. Vojtěchovský and I. M. Wanless, Cycle structure of autotopisms of quasigroups and Latin squares, *J. Combin. Des.* **20** (2012) 227–263.

Monday
16.20-16.40

Edge Construction of Molecular NSSDs

Alexander Farrugia

University of Malta, Malta

alex.farrugia@um.edu.mt

A nonsingular graph with a singular deck, or NSSD, is a graph with weighted edges and no loops whose adjacency matrix is nonsingular and whose vertex–deleted subgraphs have singular adjacency matrices. A graph G is a NSSD if and only if the inverse graph G^{-1} associated with the inverse of the adjacency matrix of G exists and is a NSSD. The structure of a NSSD induces an ipso omni–insulating molecule, in which, at the Fermi energy level, conduction between two atoms i and j does not occur whenever $i = j$.

We discuss conditions for a NSSD G to remain a NSSD after increasing the weight of one of its edges by $w \in \mathbb{R} \setminus \{0\}$; this weight change may result in the addition or removal of that edge to or from G. If the NSSD G has a pair of distinct vertices

(u, v) such that G is a NSSD for any edge weight (including zero) assigned to the edge $\{u, v\}$ in G, then (u, v) is a pliable vertex pair of G. Necessary and sufficient conditions for pliable vertex pairs (u, v) in G related to the existence of certain walks between u and v in the inverse graph G^{-1} are presented. Moreover, we consider the construction of NSSDs of a fixed even order whose adjacency matrices have the same determinant. This is accomplished by systematically introducing edges of appropriate weights to the NSSD whose components are complete graphs on two vertices. Once again, the criteria ensuring that every graph in each stage of this construction is a NSSD depend on specific walks on their inverse graphs.

References:

[1] A. Farrugia, *On the Spectra and Walks of Molecular and Controllable Graphs*, Ph.D. Thesis, University of Malta (2016).

[2] A. Farrugia, J. B. Gauci and I. Sciriha, Non–Singular Graphs with a Singular Deck, *Discrete Applied Mathematics* **202** (2016) 50–57.

[3] P. W. Fowler, B. T. Pickup, T. Z. Todorova, M. Borg, and I. Sciriha, Omni–conducting and Omni–insulating Molecules, *J. Chem. Phys.* **140** (2014) 054115.

Irregular independence and irregular domination

Tuesday
17.00-17.20

Kurt Fenech

University of Malta, Malta

kurt.fenech.10@um.edu.mt

(*joint work with* Peter Borg and Yair Caro)

A set A is said to be an *irregular independent set of* G if A is an independent set of G such that any two distinct vertices in A have different degrees in G. A set D is said to be an *irregular dominating set of* G if D is a dominating set of G such that any two distinct vertices outside of D have a distinct number of neighbours in D. We discuss mainly two parameters, firstly, the size of a largest irregular independent set of G, $\alpha_{ir}(G)$, and secondly, the size of a smallest irregular dominating set of G, $\gamma_{ir}(G)$. We present sharp bounds for both parameters in terms of other basic graph parameters, such as the order n, the size m, the minimum degree δ and the maximum degree Δ of the graph. Nordhaus-Gaddum-type relationships are also obtained for both parameters.

Monday
11.20-11.40

Existence theorems for nonnegative integral matrices with given line sums

Rosário Fernandes

Universidade Nova de Lisboa, Portugal

mrff@fct.unl.pt

(*joint work with* Henrique F. da Cruz)

Let p be a positive integer and let $\mathcal{A}^{(p)}(R,S)$ be the class of nonnegative integral matrices with entries less than or equal to p, with row sum partition R, and column sum partition S. In this talk we present some necessary and sufficient conditions for $\mathcal{A}^{(p)}(R,S) \neq \emptyset$. One of these conditions is the well known Gale-Ryser theorem.

References:

[1] R. Fernandes and H.F. da Cruz, A canonical construction for nonnegative integral matrices with given line sums, *Linear Algebra and its Applications* **484** (2015) 304-321.

Tuesday
17.40-18.00

Elementary Derivation of the Hoffman-Singleton Graph and its Automorphism Group

Jay Fraser

The Open University, United Kingdom

jay.fraser@open.ac.uk

The Moore graphs are the graphs with diameter k and girth $2k+1$. Moore graphs can be shown to be regular, thus inspiring the notation (d,k)-Moore graph for a Moore graph of degree d and diameter k. Apart from the trivial cases of $d=2$ and $k=1$, there exist only three possible cases where non-trivial Moore graphs may exist. These are the $(2,3)$-Moore graph, which must be the Petersen graph, the $(2,7)$-Moore graph, which must be the Hoffman-Singleton graph, and the final case of a potential $(2,57)$-Moore graph remains open. In this talk, we present a new derivation of the fact that a $(2,7)$-Moore graph is isomporhic to the Hoffman-Singleton graph, and further use the method to count the size of the automorphism group of the Hoffman-Singleton graph and characterise its automorphisms.

The fixed score tree graph

Thursday
17.00-17.20

Julián Alberto Fresán-Figueroa

Universidad Autónoma Metropolitana, Mexico

jfresan@correo.cua.uam.mx

(*joint work with* Eduardo Rivera-Campo)

The *tree graph* of a connected graph G is the graph $T(G)$ whose vertices are the spanning trees of G, and two trees P and Q are adjacent if P can be obtained from Q by deleting an edge p of P and adding an another edge q of Q. It is easy to prove that $T(G)$ is always connected and Cummins proved that if G has a cycle, then $T(G)$ is hamiltonian. Several variations of the tree graph have been studied, mainly for the connectivity and hamiltonicity properties.

Let $\overleftrightarrow{D_n}$ be the complete symmetric digraph with vertices $v_1, v_2, \ldots, v_n$ and let $S = ((a_1, b_1), (a_2, b_2), \ldots, (a_n, b_n))$ be a sequence of pairs of non-negative integers. We define the fixed score tree graph of $\overleftrightarrow{D_n}$ with respect to S as the graph $T_S(\overleftrightarrow{D_n})$ whose vertices are the directed spanning trees of $\overleftrightarrow{D_n}$ with score S; that is the directed spanning trees $\overrightarrow{P}$ of $\overleftrightarrow{D_n}$ such that $\deg^+_{\overrightarrow{P}}(v_i) = a_i$ and $\deg^-_{\overrightarrow{P}}(v_i) = b_i$ for $i = 1, 2, \ldots, n$.

Two directed spanning trees $\overrightarrow{P}$ and $\overrightarrow{Q}$ of $\overleftrightarrow{D_n}$ are adjacent in $T_S(\overleftrightarrow{D_n})$ if there are non-incident arcs p and r of $\overrightarrow{P}$ and non-incident arcs q and s of $\overrightarrow{Q}$, such that $\overrightarrow{Q}$ can be obtained from $\overrightarrow{P}$ by deleting p and r, adding q and s, and perhaps by flipping a directed path between them.

In this talk I will show some structural properties of the graph $T_S(\overleftrightarrow{D_n})$, and the construction of a hamiltonian cycle when S corresponds to anti-directed hamiltonian paths in $\overleftrightarrow{D_n}$.

Hypohamiltonian and Hypotraceable Oriented Graphs

Tuesday
11.20-11.40

Marietjie Frick

University of Pretoria, South Africa

marietjie.frick@gmail.com

(*joint work with* Susan van Aardt and Alewyn Burger)

A digraph is *traceable* if it has a path that visits every vertex, and *hamiltonian* if it has a cycle that visits every vertex. A digraph G is *hypotraceable* (*hypohamiltonian*) if it is nontraceable (nonhamiltonian) but $G - v$ is traceable (hamiltonian) for every vertex v in G. There is a substantial literature on hypohamiltonian and hypotraceable graphs, and searching for planar ones of small order is still a popular research topic. However, until recently, results on the oriented case were scarce. USR Murty had asked in 1974 whether hypohamiltonian oriented graphs exist. Thomassen [1] constructed an infinite family of hypohamiltonian oriented graphs and asked whether planar ones exists. Grötschel, Thomassen and Wakabayashi [2] provided techniques for constructing hypotraceable oriented graphs from hypohamiltonian oriented graphs. We survey the known results on hypohamiltonian and hypotraceable oriented graphs and we present infinite families of planar hypohamiltonian and planar hypotraceable oriented graphs.

References:

[1] C. Thomassen, Hypohamiltonian graphs and digraphs, in: *Proceedings of the International Conference on the Theory and Applications of Graphs, Kalamazoo, 1976*, Springer Verlag, Berlin (1978) 557–571.

[2] M. Grötschel, C. Thomassen and Y. Wakabayashi, Hypotraceable digraphs, *J. Graph Theory* **4** (1980) 377–381.

Thursday
15.20-15.40

Equitable Colorings of l-Corona Products of Cubic Graphs

Hanna Furmańczyk

University of Gdańsk, Poland

hanna@inf.ug.edu.pl

(*joint work with* Marek Kubale)

A graph G is equitably k-colorable if its vertices can be partitioned into k independent sets in such a way that the numbers of vertices in any two sets differ by at most one. The smallest integer k for which such a coloring exists is known as the *equitable chromatic number* of G and it is denoted by $\chi_=(G)$.

In this paper the problem of determinig the equitable chromatic number for multicoronas of cubic graphs $G \circ^l H$ is studied. The problem of ordinary coloring of multicoronas of cubic graphs is solvable in polynomial time. The complexity of the equitable coloring problem is an open question for these graphs, except for the case $l = 1$. We provide some polynomially solvable cases of cubical multicoronas and give simple linear time algorithms for equitable coloring of such graphs which use at most $\chi_=(G \circ^l H) + 1$ colors in the remaining cases.

Minimal Matrices in the Bruhat Order for Symmetric (0,1)-Matrices

Tuesday
17.00-17.20

Susana Furtado

Faculdade de Economia da Universidade do Porto and CEAFEL, Portugal

sbf@fep.up.pt

(*joint work with* Henrique F. da Cruz and Rosário Fernandes)

A Bruhat order for the class of m-by-n $(0,1)$-matrices with prescribed row and column sum vectors was defined by Brualdi and Hwang (2004), extending the Bruhat order for permutation matrices. Minimal matrices for this Bruhat order were studied in that paper and in a subsequent paper by Brualdi and Deaett (2007). When restricted to the symmetric matrices, new minimal matrices may appear besides the symmetric minimal matrices for the nonrestricted Bruhat order.

In this talk we give some results related with the description of the minimal matrices for the Bruhat order on the class of symmetric $(0,1)$-matrices with given row sum vector. We start by giving some properties of these minimal matrices. We then present minimal matrices for the Bruhat order on some particular classes. Namely, we determine all the minimal matrices when the row sums are constant and equal to 3. We also describe symmetric matrices that are minimal for the Bruhat order on the class of $2k$-by-$2k$ $(0,1)$-matrices (not necessarily symmetric) with constant row sums equal to $k+1$ and identify, in terms of the term rank of a matrix, a class of symmetric matrices that are related in the Bruhat order with one of these minimal matrices. It is known (Brualdi and Deaett, 2007) that there is a unique minimal matrix (which is symmetric) for the Bruhat order on the class of $2k$-by-$2k$ $(0,1)$-matrices with constant row sums equal to k.

Resolvable configurations

Tuesday
16.40-17.00

Gábor Gévay

University of Szeged, Hungary

gevay@math.u-szeged.hu

A *combinatorial configuration* of *type* (p_q, n_k) is an incidence structure with sets $\mathcal{P}$ and $\mathcal{B}$ of objects, called *points* and *blocks*, such that the following conditions hold: (i) $|\mathcal{P}| = p$; (ii) $|\mathcal{B}| = n$; (iii) each point is incident with q blocks; (iv) each block is incident with k points. A *geometric configuration* is defined similarly, such that the sets $\mathcal{P}$ and $\mathcal{B}$ consist of points and lines, respectively, usually in a Euclidean or a real projective plane, and the incidences are the natural geometric incidences. (Instead of lines, other geometric figures, such as circles, conics, etc. can also play the role of blocks.) Let $\mathcal{C}$ be a configuration (either combinatorial or geometric). We say that $\mathcal{C}$ is a *resolvable* configuration if the blocks of $\mathcal{C}$ can be coloured in such a way that within each colour class, the blocks partition the set of points of $\mathcal{C}$. Note that this is a generalization of the notion of a resolvable block design; e.g. Kirkman's famous schoolgirl problem leads to a resolvable configuration of type $(15_7, 35_3)$. In our talk we present examples of resolvable geometric configurations (whose blocks are lines, as well as conics), discuss certain properties of them, and pose some problems; in particular: can our example above (the "*Kirkman configuration*") be realized geometrically as a configuration of points and lines?

Tuesday
12.00-12.20

Sufficient conditions for the existence of alternating Hamiltonian paths and cycles in 2-edge-coloured multigraphs

Ilan A. Goldfeder

Universidad Autónoma Metropolitana, Mexico

ilan.goldfeder@gmail.com

(*joint work with* Alejandro Contreras-Balbuena and Hortensia Galeana-Sánchez)

A 2-edge-coloured multigraph is a multigraph such that each edge has a colour, namely red or blue, and no two parallel edges have the same colour. A path (cycle) in a 2-edge-coloured multigraph is *alternating* if no two consecutive edges have the same colour and a path (cycle) is *Hamiltonian* if it visits every vertex in the multigraph. The problem of determining the existence of alternating Hamiltonian paths and cycles in 2-edge-coloured multigraphs is NP-complete and it has been studied by several authors. In this talk we will discuss some new conditions on short paths which imply the existence of alternating Hamiltonian paths and cycles.

Monday
11.20-11.40

The Edge Intersection Graphs of Paths on a Grid

Martin Charles Golumbic

University of Haifa, Israel

golumbic@cs.haifa.ac.il

In this lecture, we will survey the mathematical and algorithmic results on the *edge intersection graphs of paths in a grid* (EPG) together with several restrictions on the representations. Two important restrictions that are motivated by network and circuit design problems are (1) allowing just a single bend in any path, and (2) limiting the paths within a rectangular grid with the endpoints of each path on the boundary of the rectangle.

Golumbic, Lipshteyn and Stern introduced EPG graphs in 2005, proving that every graph is an EPG graph, and then turning their attention to the subclass of graphs that admit an EPG representation in which every path has at most a single bend, called B_1-EPG graphs. They proved that any tree is a B_1-EPG graph and gave a structural property that enables generating non B_1-EPG graphs. A characterization of the representation of cliques and chordless 4-cycles in B_1-EPG graphs was given, and it was also proved that single bend paths on a grid have Strong Helly number 4, and when the paths satisfy the usual Helly property, they have Strong Helly number 3. Subsequent results by our colleagues will be surveyed, as well as open problems and future work.

We then present our new work on boundary generated B_1-EPG graphs together with Gila Morgenstern and Deepak Rajendraprasad. For two boundary vertices u and v on two adjacent boundaries of a rectangular grid $\mathcal{G}$, we call the unique single-bend path connecting u and v in $\mathcal{G}$ using no other boundary vertex of $\mathcal{G}$ as the path *generated* by (u, v). A path in $\mathcal{G}$ is called *boundary-generated*, if it is generated by some pair of vertices on two adjacent boundaries of $\mathcal{G}$. In this work, we study the edge-intersection graphs of boundary-generated paths on a grid or B_1-EPG graphs.

We show that B_1-EPG graphs can be covered by two collections of vertex-disjoint cobipartite chain graphs. This leads us to a linear-time testable characterization of B_1-EPG trees and also a tight upper bound on the equivalence covering number of general B_1-EPG graphs. We also study the cases of two-sided B_1-EPG and three-sided B_1-EPG graphs, which are respectively, the subclasses of B_1-EPG graphs obtained when all the boundary vertex pairs which generate the paths are restricted to lie on at most two or three boundaries of the grid. For the former case, we give a complete characterization.

We do not know yet whether one can efficiently recognize B_1-EPG graphs. Though the problem is linear-time solvable on trees, we suspect that it might be NP-hard in general.

On the clique number of a strongly regular graph

Wednesday
11.40-12.00

Gary Greaves

Nanyang Technological University, Singapore

gary@ntu.edu.sg

(*joint work with* Leonard H. Soicher)

In this talk I will present new upper bounds for the clique numbers of strongly regular graphs in terms of their parameters. I will show how we improve on the Delsarte bound for infinitely many feasible parameter tuples for strongly regular graphs, including infinitely many parameter tuples that correspond to Paley graphs.

Tuesday
12.20-12.40

Maltese mathematics in the 16$^{\text{th}}$ and 17$^{\text{th}}$ century

Harald Gropp

Universitaet Heidelberg, Germany

d12@ix.urz.uni-heidelberg.de

The discussion of Maltese mathematics in the early modern period will be, apart from a short introduction into Maltese history, focused on two examples, the Maltese contribution to the Gregorian calendar reform of 1584 and the stay of the Jesuit Athanasius Kircher in Malta in 1637/38 and his activities concerning Arabic manuscripts as a later professor of mathematics and oriental languages in Roma.

Tuesday
16.20-16.40

Orbital matrices and configurations with natural index

Harald Gropp

Universitaet Heidelberg, Germany

d12@ix.urz.uni-heidelberg.de

In the first Malta conference in 1990 I introduced and discussed configurations with natural index. Configurations are linear regular uniform hypergraphs. The index of a configuration is the proportion of the degrees of regularity and uniformity. The second topic which will be discussed are orbital matrices. These are generalizations of incidence matrices of symmetric designs where the entries may be also larger than only 0 and 1.

Thursday
17.20-17.40

Parent-identifying set systems

Yujie Gu

University of Tsukuba, Japan

s1530147@u.tsukuba.ac.jp

(*joint work with* Ying Miao)

Traitor tracing in broadcast encryption was introduced by Chor et al. in [1,2]. Stinson et al. [3] proposed the traceability scheme (TS), based on a threshold secret sharing scheme, and studied it from a combinatorial viewpoint. Parent-identifying set system was investigated in [4] with the advantage that can accommodate more users than TS.

A (w, v) t-parent-identifying set system (or t-IPPS(w, v), for short) is a pair $(\mathcal{X}, \mathcal{B})$ such that $|\mathcal{X}| = v$, $\mathcal{B} \subseteq \binom{\mathcal{X}}{w}$, with the property that for any w-subset $T \subseteq \mathcal{X}$, either $P_t(T)$ is empty, or

$$\bigcap_{\mathcal{P} \in P_t(T)} \mathcal{P} \neq \emptyset,$$

where

$$P_t(T) = \{\mathcal{P} \subseteq \mathcal{B} : \ |\mathcal{P}| \leq t, \ T \subseteq \bigcup_{B \in \mathcal{P}} B\}.$$

Collins [4] showed that the upper bound for the number of blocks in a t-IPPS(w, v) is $O(v^{\lceil \frac{w}{\lfloor t^2/4 \rfloor + \lceil t/2 \rceil} \rceil})$. In this talk, first, we give an improvement for this by showing that the upper bound for t-IPPS(w, v) is $O(v^{\lceil \frac{w}{\lfloor t^2/4 \rfloor + t} \rceil})$, which is realized by analyzing the minimum size of own-subsets possessed by some blocks in a t-IPPS. Next, by using the expurgation method, we prove that for fixed t, w and sufficiently large v, there exists a t-IPPS(w, v) with size $O(v^{\frac{w}{\lfloor t^2/4 \rfloor + t}})$, which has the same order with the new upper bound when $\lfloor t^2/4 \rfloor + t$ is a divisor of w.

References:

[1] B. Chor, A. Fiat and M. Naor, Tracing traitors, *in Cryto'94 (Lecture Notes in Computer Science),* Berlin, Heidelberg, New York: Springer-Verlag, **839** (1994) 480–491.

[2] B. Chor, A. Fiat, M. Naor and B. Pinkas, Tracing traitors, *IEEE Trans. Inf. Theory* **46** (2000) 893–910.

[3] D. R. Stinson and R. Wei, Combinatorial properties and constructions of traceability schemes and frameproof codes, *SIAM J. Discrete Math.* **11** (1998) 41–53.

[4] M. J. Collins, Upper bounds for parent-identifying set systems, *Des. Codes Cryptogr.* **51** (2009) 167–173.

The distance Laplacian matrix of graphs

Thursday
11.20-11.40

Pierre Hansen

GERAD and HEC Montréal, Canada

pierre.hansen@gerad.ca

(*joint work with* Mustapha Aouchiche)

The distance Laplacian of a connected graph G is defined by $\mathcal{D}^L = Diag(Tr) - \mathcal{D}$, where $\mathcal{D}$ is the distance matrix of G, and $Diag(Tr)$ is the diagonal matrix whose main entries are the vertex transmissions in G. The spectrum of $\mathcal{D}^L$ is called the distance Laplacian spectrum of G [1]. In the present talk, we investigate some properties of the distance Laplacian spectrum. We show, among other results, equivalence between the distance and the distance Laplacian spectra over the class of transmission regular graphs, also between the Laplacian and the distance Laplacian spectra over the class of graphs with diameter 2, and similarities with the algebraic connectivity [1,2].

References:

[1] M. Aouchiche and P. Hansen, Two Laplacians for the distance matrix of a graph, *Linear Algebra Appl.* **439** (2013) 21–33.

[2] M. Aouchiche and P. Hansen, Some properties of the distance Laplacian eigenvalues of a graph, *Czechoslovak Math. Jour.* **64 (139)** (2014) 751–761.

Monday
17.40-18.00

Extremal values in graphs for metric-locating-dominating partitions

Carmen Hernando

Universitat Politècnica de Catalunya, Spain

carmen.hernando@upc.edu

(*joint work with* Mercè Mora, Ignacio M. Pelayo)

Let $G = (V, E)$ be a graph of order n. Let $\Pi = \{S_1, \ldots, S_k\}$ be a partition of V. We denote by $r(u|\Pi)$ the vector of distances between a vertex $v \in V$ and the elements of Π, that is, $r(v|\Pi) = (d(v, S_1), \ldots, d(v, S_k))$.

The partition Π *dominates* G if, for every $i \in \{1, \ldots, k\}$ and for every vertex $v \in S_i$, $d(v, S_j) = 1$, for some $j \neq i$. The *partition domination number* $\gamma_p(G)$ equals the minimum cardinality of a dominating partition in G.

The partition Π is called a *locating partition* of G if, for any pair of distinct vertices $u, v \in V$, $r(u, \Pi) \neq r(v, \Pi)$. The *partition dimension* $\beta_p(G)$ of G is the minimum cardinality of a locating partition of G.

The partition Π is called a *metric-locating-dominating partition* of G if it is both dominating and locating. The *partition metric-location-domination number* $\eta_p(G)$ of G is the minimum cardinality of a metric-locating-dominating partition of G.

The partition Π is called a *neighbor-locating-dominating partition* of G if, for every $i \in \{1, \ldots, k\}$ and for every pair of distinct vertices $v, w \in S_i$, there exists $j \in \{1, 2, \ldots, k\}$ such that $d(v, S_j) = 1$ and $d(w, S_j) > 1$. The *partition neighbor-location-domination number* $\lambda_p(G)$ of G is the minimum cardinality of a neighbor-locating-dominating partition of G.

Parameters η_p and λ_p have been introduced and studied in [3]. Among other results, we have obtained tight bounds for these parameters in terms of the order of the graph and we have characterized all graphs attaining them. Furthermore, we have generalized for these two parameters some well-known properties in other related parameters, with the approach given in [1,2].

References:

[1] J. Cáceres, C. Hernando, M. Mora, I. M. Pelayo, M. L. Puertas, Locating-dominating codes: Bounds and extremal cardinalities, *Applied Mathematics and Computation* **220** (2013) 38–45.

[2] C. Hernando, M. Mora and I. M. Pelayo, Nordhaus-Gaddum bounds for locating domination, *European Journal of Combinatorics* **36** (2014) 1-6.

[3] C. Hernando, M. Mora and I. M. Pelayo, Extremal values for metric-locating-dominating partition in graphs, preprint.

Several families with incomparability and complementarity conditions

Wednesday
11.20-11.40

Anthony J. W. Hilton

University of Reading, UK

a.j.w.hilton@reading.ac.uk

(*joint work with* John L. Goldwasser and Jie Zheng)

Two sets are *weakly incomparable* if neither properly contains the other; they are *strongly incomparable* if they are unequal and neither properly contains the other. Two families $\mathcal{A}$ and $\mathcal{B}$ of sets are weakly (or strongly) incomparable if no set in one of $\mathcal{A}$ and $\mathcal{B}$ is weakly (or strongly) comparable to a set in the other. A family $\mathcal{A}$ of sets is uncomplemented if $\mathcal{A}$ contains no subset and its complement. We show that the following two statements are equivalent (as either can be deduced from the other).

(1) If $\mathcal{A}_1, \ldots, \mathcal{A}_t$ are uncomplemented, mutually weakly incomparable families of subsets of an n-set, then

$$|\mathcal{A}_1| + \ldots + |\mathcal{A}_t| \leq \max\left(2^{n-1}, t\begin{pmatrix} n \\ \lfloor \frac{n}{2} \rfloor + 1 \end{pmatrix}\right).$$

(2) If $\mathcal{A}_1, \ldots, \mathcal{A}_t$ are uncomplemented, mutually strongly incomparable families of subsets of an n-set, then

$$|\mathcal{A}_1| + \ldots + |\mathcal{A}_t| \leq 2^{n-1}.$$

Both these relate to a conjecture of Hilton made in 1976, reported in a Math. Review article by D.J. Kleitman.

We also show that if $\mathcal{A}_1, \ldots, \mathcal{A}_t$ are mutually weakly incomparable families of subsets of an n-set, and if they are mutually uncomplemented (i.e. if $A \in \mathcal{A}_j$ then $\overline{A} \notin \mathcal{A}_i$ if $i \neq j$) then

$$|\mathcal{A}_1| + \ldots + |\mathcal{A}_t| \leq \max\left(2^n, t\binom{n}{\lfloor \frac{n}{2} \rfloor + 1}\right).$$

We also show that if $\mathcal{A}_1, \ldots, \mathcal{A}_6$ are t mutually uncomplemented Sperner families of subsets of an n-set then:

(1) If n is odd, then

$$|\mathcal{A}_1| + \ldots + |\mathcal{A}_t| \leq t\binom{n}{\lfloor \frac{n}{2} \rfloor}.$$

(2) If n is even, then

$$|\mathcal{A}_1| + \ldots + |\mathcal{A}_t| \leq \binom{n}{\frac{n}{2}} + (t-1)\binom{n}{\frac{n}{2}+1}.$$

Friday
12.20-12.40

Complexity results on dominating codes

Olivier Hudry

Télécom ParisTech, France

olivier.hudry@telecom-paristech.fr

(*joint work with* Antoine Lobstein)

Given an undirected, connected, graph $G = (V, E)$, a subset C of V is said to be a *dominating code* of G if any vertex belongs to C or admits a neighbour belonging to C. This can be extended, for any positive integer r, to r-dominating codes of G: C is said to be an r-dominating code (or simply r-DC) of G if, for any vertex v of G, there exists a vertex x (possibly v itself) belonging to C such that the distance between v and x is at most r, where the considered distance is the usual distance provided by a shortest path between v and x in G.

A usual problem, arising from combinatorial optimization in graphs, consists in minimizing the size of an r-DC. The decision problem associated with this optimization problem is known to be NP-complete for $r = 1$. We investigate the complexity of several problems linked with domination in graphs, for any positive r:

• the computation of the minimum size of an r-DC;

• the search of an optimal r-DC;

• the existence and the computation of an optimal r-DC containing a prescribed subset of vertices (also known as "membership problems").

We show that the computation of the minimum size of an r-DC belongs to the complexity class called FL^{NP} (or also $F\Theta_2$) and is L^{NP}-hard (or also Θ_2-hard;

remember that this means, broadly speaking, that we can solve this problem thanks to an algorithm which solves an NP-complete problem, by applying it a logarithmic number of times, and that the considered problem is among the hardest ones with such a property). Similarly, we show that the problem of the existence of an r-DC containing a prescribed subset of vertices is L^{NP}-complete (or Θ_2-complete), while, for the search of optimal solutions, we show that this problem belongs to FP^{NP} (also called $F\Delta_2$; remember that this means, broadly speaking, that we can solve this problem thanks to an algorithm which solves an NP-complete problem, by applying it a polynomial number of times) and that it is L^{NP}-hard (see [1] for more details).

References:

[1] O. Hudry and A. Lobstein, More Results on the Complexity of Domination Problems in Graphs, *International Journal of Information and Coding Theory*, to appear.

Pebbling in Chordal Graphs

Thursday
15.20-15.40

Glenn Hurlbert

Virginia Commonwealth University, U.S.A.

ghurlbert@vcu.edu

(*joint work with* Liliana Alcón and Marisa Gutierrez)

Graph pebbling is a network model for transporting discrete resources that are consumed in transit. Deciding whether a given configuration on a particular graph can reach a specified target is NP-complete, even for diameter two graphs, and deciding whether the pebbling number has a prescribed upper bound is Π_2^{P}-complete. It has been conjectured that calculating the pebbling number of a chordal graph of bounded diameter or pathwidth can be done in polynomial time. Recently we proved this for split graphs, 2-paths, and semi-2-trees. We will discuss these results and the important tools developed to attack such problems.

Monday
16.20-16.40

The largest t-intersecting Erdős-Ko-Rado sets of polar spaces

Ferdinand Ihringer

University of Regina, Canada

Ferdinand.Ihringer@uregina.ca

(*joint work with* Klaus Metsch)

A t-intersecting *Erdős-Ko-Rado set* (EKR set) Y of $\{1, \ldots, n\}$ is a family of k-sets which pairwise intersect non-trivially. A famous result due to Ahlswede and Khachatrian from 1997 showed that, besides a few special cases, the largest t-intersecting families are of the form

$$\{A : |A| = k \text{ and } |A \cap \{1, \ldots, 2r + t\}| \geq r + t\}$$

for some r. These results were generalized to various structures such as various permutation groups or vector spaces.

If we equip a vector space over a finite field of order q with a reflexive, non-degenerate sesquilinear form, then the subspaces that vanish on this form constitute a highly symmetric geometric structure, a polar space. We will discuss t-intersecting EKR results for various finite classical polar spaces. In particular we will present a classification result for the largest t-intersecting EKR sets of maximal totally isotropic subspaces for a polar space of rank d and $d - t \in O(\sqrt{d})$.

Thursday
15.40-16.00

Bridging the gap between sparse and dense

Mikhail Isaev

UNSW, Australia

isaev.m.i@gmail.com

(*joint work with* Brendan D. McKay)

The asymptotic number of d-regular graphs on n vertices is known for ranges $d = o(n^{1/2})$ and $c\frac{\log n}{n} < d < n/2$. These results (obtained by B.D. McKay and N.C. Wormald in 1990 and 1991) are strongly distinguished by the type of mathematics used to solve them, which is combinatorial in the sparse range and complex-analytic in the dense range. Our new approach based on cumulant expansions allowed us to significantly enlarge the range of complex-analytic methods to make it overlap with the sparse range. It also applies to many other similar enumeration problems in combinatorics.

Number of maximum independent sets on trees

Thursday
17.40-18.00

Daniel A. Jaume

Universidad Nacional de San Luis, Argentina

djaume@unsl.edu.ar

In 1986 Wilf proved that the largest number of maximal independent sets in a tree is $1 + 2^{\frac{n}{2}-1}$ for $n \geq 2$ and even, and $2^{\frac{n-1}{2}}$ for n odd, see [3]. In 1988 Sagan, [2], gave a short graph-theoretic proof of the Wilf result and characterized the extremal trees. Later, Wilf posed the following question: Which is the greatest number of maximum independent sets for a tree T of order n? In 1991 Zito, in her work [3], proved that the greatest number of maximum independent sets for a tree T of order n is

$$\begin{cases} 2^{\frac{n-3}{2}} & \text{for odd } n > 1, \\ 1 + 2^{\frac{n-2}{2}} & \text{for even } n. \end{cases}$$

In this work, by using the null decomposition of trees introduced by Jaume and Molina in [1], we prove that for any tree T

$$\nu(T) = \alpha(T) - \operatorname{null}(T)$$

where $\nu(T)$ is the matching number of T, $\alpha(T)$ is the independence number of T, and $\operatorname{null}(T)$ is the dimension of the null space of the adjacency matrix of T. Furthermore we prove that for any tree T

$$a(T) = \prod_{N \in \mathcal{F}_N(T)} a(N)$$

where $a(T)$ is the number of maximum independent sets of T, and $\mathcal{F}_N(T)$ is the N-forest of T, see [1]. This last formula allows us to build parallelizable algorithms in order to enumerate and find all the maximum independent sets on any tree.

References:

[1] D.A. Jaume and G. Molina, Null decomposition of trees, submitted (2016).

[2] B.E. Sagan, A note on independent sets in trees, *SIAM Journal on Discrete Mathematics* **1** (1988) 105–108.

[3] H. S. Wilf, The number of maximal independent sets in a tree, *SIAM Journal on Algebraic Discrete Methods* **7** (1986) 125–130.

[4] J. Zito, The structure and maximum number of maximum independent sets in trees, *Journal of Graph Theory* **15** (1991) 207–221.

Tuesday
16.40-17.00

Total graph coherent configurations: new graphs from large Moore graphs

Leif K. Jørgensen

Aalborg University, Denmark

leif@math.aau.dk

(*joint work with* Mikhail Klin and Matan Ziv-Av)

For a graph Γ, the total graph $T(\Gamma)$ has vertex set $V(\Gamma) \cup E(\Gamma)$ and adjacency in $T(\Gamma)$ means adjacency/incidence in Γ. The automorphism group G of $T(\Gamma)$ is usually isomorphic to the automorphism group of Γ. The edge set of the complete graph with vertex set $V(T(\Gamma))$ is partitioned in orbits under the action of G. (Alternatively, we may consider a coarser, combinatorially defined partition, called the coherent configuration generated by $T(\Gamma)$.) Our goal is to construct a new graph with vertex set $V(T(\Gamma))$ and edge set a union of some of the orbits, and with automorphism group larger than G. A purely combinatorial alternative is to get a coherent configuration of small rank, generated in some sense by $T(\Gamma)$.

In particular we consider the case when Γ is the complement of a Moore graph. In this talk we will focus on the case where Γ is the complement of the Hoffman-Singleton graph. We then get nice graphs (in fact a 4 class association scheme) of order 1100 with the Higman-Sims group as a subgroup of index 2 in the full automorphism group of order 88704000. We will explain a connection between the Hoffman-Singleton graph, a strongly regular graph on 100 vertices and the graphs on 1100 vertices.

We will also discuss the case when Γ is the complement of a putative Moore graph of valency 57.

Dynamic Graph Outerplanarity in linear worst case time

Thursday
17.00-17.20

Claudia Justel

Instituto Militar de Engenharia, Brazil

cjustel@ime.eb.br

(*joint work with* Geraldo Avelino)

There is a large variety of problems modeled by graphs, for which the associated graph changes in time. For instance, dynamic algorithms for graphs which explore properties such as connectivity, minimum spanning tree and planarity ([2],[3],[4]) are well known. In this work, we implement a C++ dynamic algorithm (insertion and deletion of edges are supported) to maintain outerplanarity as in [1].

Consider an unweighted and undirected simple graph G. The fully dynamic outerplanarity problem is defined as follows: Let $G = (V, E)$ be an outerplanar graph, where V is a fixed set of n vertices and an online sequence of updates (insertion or deletion of edges $e \notin E$). We ask queries of the following form: Is the graph $G + e$ outerplanar? Is the graph $G - e$ outerplanar? The algorithm in [1] maintains the embedding of the outerplanar graph during insertions or deletions of edges. Two levels of information on the graph are stored, one with data for the graph and the other with data for the block-connection graph. The block-connection graph, $BC(G)$, is defined as follows: the vertices of $BC(G)$ are the connections (maximal subgraphs of G that are trees or articulations in G) and the blocks (maximal biconnected subgraphs of G); edges are links between vertices in $BC(G)$ that have a common vertex in G, and are not simultaneously blocks or connections; each edge in $BC(G)$ has a label given by the common vertex in G. With this information in hand it is possible to maintain the embedding of the outerplanar graph after a sequence of updates. The correctness is ensured by the results in [1] and the worst case complexity $O(n)$ is maintained. The algorithm was implemented in C++, using its Standard Template Library (STL). Experiments performed with an Intel Core i5, 2.5GHz, 4GB RAM computer show good performance.

References:

[1] L. H. C. Araujo, *Algoritmos Dinâmicos para Manutenção de Grafos Cordais e Periplanares*, Ph.D. Thesis, Universidade Federal do Rio de Janeiro (2004).

[2] J. Holm, K. Lichtenberg, M. Thorup, *Poly-logarithmic deterministic fully-dynamic algorithms for connectivity, minimum spanning tree, 2-edge, and biconnectivity*, *J. ACM* **48** (2001) 723–760.

[3] G. Italiano, Z. Galil, N. Sarnak, *Fully dynamic planarity testing with applications*, *J. ACM* **46:1** (1999) 28–91.

[4] B.M. Kapron,V. King, B. Mountjoy, *Dynamic graph connectivity in polylogarithmic worst case time. ACM-SIAM* (2013) 1131–1142.

Monday
16.00-16.20

Symmetry Breaking in Graphs and the 1-2-3 Conjecture

Rafał Kalinowski

AGH University, Poland

kalinows@agh.edu.pl

(*joint work with* Monika Pilśniak and Mariusz Woźniak)

This talk brings together two concepts in the theory of graph colourings: edge or total colourings distinguishing adjacent vertices and those breaking symmetries of a graph. Let

$$c : E(G) \to \{1, \ldots, k\}$$

be an edge colouring, not necessarily proper, of a graph G.

A colouring c *breaks an automorphism* φ of G if φ does not preserve c. The smallest k for which there exists a c breaking all non-trivial automorphisms is called the *distinguishing index* of a graph G, and is denoted by $D'(G)$. For connected graphs of order at least six, a sharp upper bound $D'(G) \leq \Delta(G)$ was obtained by Kalinowski and Pilśniak.

We say that c is *neighbour-distinguishing by sums* if, for every pair u, v of adjacent vertices

$$\sum_{e \ni u} c(e) \neq \sum_{e \ni v} c(e).$$

Karoński, Łuczak and Thomason formulated the 1-2-3 Conjecture that every graph admits such a colouring for $k = 3$. This conjecture has been confirmed for some classes of graphs, but in general it remains open since 2004. Up to now, the best result for $k = 5$ is due to Kalkowski, Karoński and Pfender.

In this talk, we introduce a class of automorphisms such that edge colourings breaking them are connected to edge colourings distinguishing neighbours by sums. We call an automorphism φ of a graph G *small* if there exists a vertex of G that is mapped by φ onto its neighbour. The *small distinguishing index of* G, denoted $D'_s(G)$, is the least k such that there exists a c breaking all small automorphisms of G. We prove that $D'_s(G) \leq 3$ for every graph G without K_2 as a component, thus supporting, in a sense, the 1-2-3 Conjecture.

We also consider an analogous problem for total colourings in connection with the 1-2 Conjecture of Przybyło and Woźniak.

On Chvátal's conjecture and Erdős–Ko–Rado Graphs

Monday
16.00-16.20

Vikram Kamat

Virginia Commonwealth University, U.S.A.

vkamat@vcu.edu

(*joint work with* Peter Borg, Éva Czabarka and Glenn Hurlbert)

A fundamental theorem of Erdős, Ko and Rado states that the size of a family of pairwise intersecting r-subsets of $[n] = \{1, \dots, n\}$, when $r \leq n/2$, is at most $\binom{n-1}{r-1}$, with equality holding in the case $r < n/2$ if and only if the family is a collection of all r-subsets containing a fixed element. In this talk, we focus our attention on a longstanding conjecture of Chvátal that aims to generalize the EKR theorem for *hereditary* set systems and another closely-related conjecture of Holroyd and Talbot pertaining to a graph-theoretic generalization of the EKR theorem for independent sets in graphs. We present a result that verifies Chvátal's conjecture for hereditary families containing sets of size at most 3, and also multiple results that verify the Holroyd-Talbot conjecture and its variants for certain graph classes.

Complexity questions for minimally t-tough graphs

Monday
12.00-12.20

Gyula Y. Katona

Budapest University of Technology and Economics, Hungary

kiskat@cs.bme.hu

(*joint work with* Kitti Varga, István Kovács)

A graph G is minimally t-tough if the toughness of G is t and the deletion of any edge from G decreases the toughness. Kriesell conjectured that for every minimally 1-tough graph the minimum degree $\delta(G) = 2$. In the present talk we investigate different complexity questions related to this conjecture.

First we show that recognizing minimally t-tough graphs is a hard task for some t values. It is a DP-complete problem (implying that is probably even harder than being NP-hard). Does this change if the question is asked for some special graph classes like chordal, split, claw-free and $2K_2$-free graphs and for special t values? The answers vary. In some cases there are no such graphs at all, so it is really easy to recognize them. In some other cases, we can characterize all the graphs. Yet in some particular case we can at least recognize it in polynomial time. Many open questions remain.

Monday
17.20-17.40

On the transmission non-regularity of graphs

Ezgi Kaya

Iğdir University, Turkey

ezgi.kaya@igdir.edu.tr

(*joint work with* A. Dilek Maden)

Let G be a graph. The distance between v_i and v_j in G is defined as the length of a shortest path between them. The transmission of a vertex u, denoted by $T_G(u)$, is the sum of distances from it to all the other vertices in the graph G, i.e., $T_G(u) = \sum_{v \in V(G)} d_G(u,v)$. Therefore, the Wiener index of a graph G is equal to half the sum of the transmissions of all vertices in the graph G. The definition of the Co-PI index can be represented in the following way: $Co - PI_v(G) = \sum |T_G(u) - T_G(v)|$, where the summation goes over all edges of G. G is said to be transmission-regular if all its vertices have the same transmission. If not, G is said to be transmission-non-regular. For a transmission-non-regular graph, how can we measure the extent of its transmission-non-regularity? Because the Co-PI index only scans edges in a graph, it will remain inadequate to measure the transmission-non-regularity of a graph. So, in this study we define the transmission-non-regularity index as $NT(G) = \sum |T_G(u) - T_G(v)|$, where the summation goes over all vertices of G. Also, we present some bounds for the transmission index $NT(G)$ of graphs.

Monday
11.20-11.40

Edge-girth-regular graphs

György Kiss

Eötvös Loránd University, Hungary
and
University of Primorska, Slovenia

kissgy@cs.elte.hu

(*joint work with* Robert Jajcay and Štefko Miklavič)

We consider a new type of regularity we call edge-girth-regularity. An edge-girth-regular (v, k, g, λ)-graph Γ is a k-regular graph of order v and girth g in which every edge is contained in λ distinct g-cycles. This concept is a generalization of the well-known (v, k, λ)-edge-regular graphs (that count the number of triangles) and appears in several related problems such as Moore graphs and Cage and Degree/Diameter Problems. All edge- and arc-transitive graphs are edge-girth-regular as well. We derive a number of basic properties of edge-girth-regular graphs, systematically consider cubic and tetravalent graphs from this class, and introduce several constructions that produce infinite families of edge-girth-regular graphs. We also exhibit several surprising connections to regular embeddings of graphs in orientable surfaces.

Target Set Selection in Dense Graph Classes

Thursday
16.00-16.20

Dušan Knop

Charles University, Prague, Czech Republic

knop@kam.mff.cuni.cz

(*joint work with* Pavel Dvořák and Tomáš Toufar)

We study the Target Set Selection problem (TSS for short), introduced by Kempe et al. [1], from the area of computational social choice from a parameterized complexity perspective and give a characterization of the target sets in some graph classes.

Target Set Selection. Let $G = (V, E)$ be a graph, $S \subseteq V$, and $f \colon V \to \mathbf{N}$ a *threshold function*. The *activation process* arising from the set $S_0 = S$ is an iterative process with resulting sets $S_0, S_1, \ldots$ such that for $i \geq 0$

$$S_{i+1} = S_i \cup \{v \in V : |N(v) \cap S_i| \geq f(v)\},$$

where by $N(v)$ we denote the set of vertices adjacent to v. We say that the set S is a *target set* and the activation process $S = S_0, \ldots, S_n$ is *successful* if $S_n = V$. We speak about the Majority TSS problem (MajTSS for short) if the threshold function f fulfils $f(v) = \lceil N(v)/2 \rceil$.

Our Results.

- There is an fpt-algorithm for the MajTSS problem parameterized by the neighborhood diversity of the input graph.
- The TSS problem is W[1]-hard parameterized by the neighborhood diversity of the input graph.
- There is an fpt-algorithm for the MajTSS problem parameterized by the size of the twin cover.
- The MajTSS problem is W[1]-hard parameterized by the modular-width of the input graph.

References:

[1] D. Kempe, J. Kleinberg, and É. Tardos, Maximizing the spread of influence through a social network, *Proceedings of the ninth ACM SIGKDD international conference on Knowledge discovery and data mining* (2003) 137–146.

[2] P. Dvořák, D. Knop, and T. Toufar, Target Set Selection in Dense Graph Classes, *CoRR* **abs/1610.07530** (2016), 1–19.

Tuesday
11.40-12.00

On small strength two covering arrays

Janne I. Kokkala

Aalto University, Finland

janne.kokkala@aalto.fi

(*joint work with* Karen Meagher, Reza Naserasr, Kari J. Nurmela, Patric R. J. Östergård and Brett Stevens)

A *covering array* $\mathrm{CA}(N; t, k, v)$ is an $N \times k$ array over a set of size v such that in every $N \times t$ subarray, each t-tuple of symbols occurs as a row at least once. It is said that a $\mathrm{CA}(N; t, k, v)$ has *size* N, *strength* t, k *factors*, and v *levels*. A *uniform* covering array, $\mathrm{UCA}(N; t, k, v)$, is a covering array in which in every column, every symbol occurs either $\lceil N/v \rceil$ or $\lfloor N/v \rfloor$ times. A covering array is said to be *optimal* if the size N is the smallest possible for given t, k, v.

In this work, we study the structure of small covering arrays with strength two, that is, $t = 2$. Theoretical considerations and previously known covering arrays suggest that many small optimal covering arrays may be uniform. In particular, for all sets of parameters N, k, v for which optimal strength two covering arrays are known, at least one of them is uniform. In some cases, all known optimal covering arrays are uniform. To gain more understanding, we also perform an exhaustive computational classification of covering arrays with some parameter sets.

Friday
11.40-12.00

The complexity of minimum-length path decompositions

Wieslaw Kubiak

Memorial University of Newfoundland, Canada

wkubiak@mun.ca

(*joint work with* Dariusz Dereniowski and Yori Zwols)

We consider a bicriterion generalization of the pathwidth problem: given integers k, l and a graph G, does there exist a path decomposition of G of width at most k and length (i.e., number of bags) at most l? We provide a complete complexity classification of the problem in terms of k and l for general graphs. Contrary to the original pathwidth problem, which is fixed-parameter tractable with respect to k, the generalized problem is NP-complete for any fixed $k \geq 4$, and also for any fixed $l \geq 2$. On the other hand, we give a polynomial-time algorithm that constructs a minimum-length path decomposition of width at most $k \leq 3$ for any disconnected input graph. As a by-product, we obtain an almost complete classification for connected graphs: the problem is NP-complete for any fixed $k \geq 5$, and polynomial for any $k \leq 3$.

Chromatic Sum for Total Colorings of Graphs

Monday
12.00-12.20

Ewa Kubicka

University of Louisville, USA

ewa@louisville.edu

(*joint work with* Grzegorz Kubicki and Maxfield Leidner)

Consider a proper coloring ϕ of vertices of a graph G using natural numbers; i.e. $\phi : V(G) \to N$ and $\phi(u) \neq \phi(v)$ whenever uv is an edge of G. The *chromatic sum* of G, denoted $\Sigma(G)$, is the minimum sum $\sum_{v\in V(G)} \phi(v)$ taken over all proper colorings ϕ of G. A coloring is *optimal* if the sum of colors equals $\Sigma(G)$. This idea was introduced by Kubicka [2] in 1989. Erdös, Kubicka, and Schwenk [1] constructed infinite families of graphs for which the minimum number of colors necessary to get an optimal coloring of G was larger than $\chi(G)$. This graph parameter, the minimum number of colors necessary for an optimal coloring, is called the *strength* of G and is denoted by $\sigma(G)$. In [1], it is shown that even trees can have arbitrarily high strength, even though their chromatic number is 2. In fact, Erdös, Kubicka, and Schwenk [1] found for every $k \geq 3$ the smallest tree of strength k. We say that a graph G is *strong* if $\chi(G) < \sigma(G)$. The smallest strong graph is the tree on eight vertices and it was introduced in [2]. These color-sum concepts were applied to edge coloring as well. In an analogous way, one can define the *edge chromatic sum* of a graph, its edge strength σ', and ask the question of whether or not $\chi' = \sigma'$. In 1997, Mitchem, Morris, and Schmeichel [3] proved that every graph has a proper edge coloring with minimum sum that uses only Δ or $\Delta + 1$ colors, where Δ denotes the largest degree of a graph. We say that a graph G with this property, namely $\chi'(G) < \sigma'(G)$, is *E-strong*. In the same paper, Mitchem et al. [3] provide infinite families of E-strong graphs. We define similar concepts for total colorings of graphs. The *total chromatic sum* of a graph is the minimum sum of colors (natural numbers) taken over all proper colorings of vertices and edges of a graph. We construct infinite families of graphs for which the minimum number of colors to achieve the total chromatic sum is larger than the total chromatic number.

References:

[1] P. Erdős, E. Kubicka and A. Schwenk, Graphs that require many colors to achieve their chromatic sum, *Congressus Numerantium* **71** (1990) 17-28.

[2] E. Kubicka, The chromatic sum and efficient tree algorithms, *Ph.D. Thesis, Western Michigan University* (1989) 149 pages.

[3] J. Mitchem, P. Morris, and E. Schmeichel, On the cost chromatic number of outerplanar, planar, and line graphs, *Discussiones Mathematicae Graph Theory* **2** (1997) 229-241.

Monday
15.40-16.00

Boundary-type sets in maximal outerplanar graphs

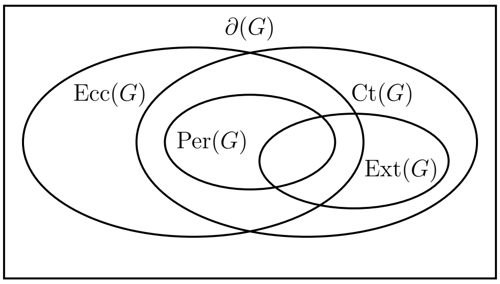
Grzegorz Kubicki

University of Louisville, U.S.A. and WSB University in Gdansk, Poland

gkubicki@louisville.edu

(*joint work with* Benjamin Allgeier)

Using the distance between vertices in a connected graph, different boundary-type sets were defined and investigated for graphs; see [1], [2] and [3].
The **periphery** Per(G) of a graph G is the set of vertices of maximum eccentricity. The **extreme set** Ext(G) of G is the set off all its extreme (also called simplicial) vertices; an extreme vertex is such that its neighborhood induces a complete graph. A contour vertex has the eccentricity larger than or equal to the eccentricities of its neighbors. The **contour** Ct(G) of G is the set of all its contour vertices. The **eccentricity** Ecc(G) of a graph G is the set of all its eccentric vertices, i.e. vertices that are antipodal to some other vertex in G. A vertex v is a boundary vertex if there is another vertex u in G such that no u-v geodesic can be extended at v to a longer geodesic. The **boundary** $\partial(G)$ of G is the set of all its boundary vertices. Basic containments between these sets are depicted in the figure below.



A graph G is **outerplanar** if there exists a plane graph G' isomorphic to G such that every vertex of G' lies on the boundary of the exterior region. An outerplanar graph is **maximal outerplanar** (or MOP for short) if adding any edge results in a graph that is not outerplanar. A MOP is also a triangulation of a polygon. MOPs can be viewed also as generalizations of trees. They form a subfamily of 2-trees, because one can construct a MOP from another MOP G by adding a new vertex and making it adjacent to both vertices of any outer edge of G.

The boundary-type sets are rather trivial for trees and are all subsets of the leaves. We provide a characterization of $\partial(G)$, Ct(G), and Ext(G) for the family of MOPs. We show that, unlike for trees, all containments in the Venn diagram are proper for MOPs, a subfamily of 2-trees.

References:

[1] J. Cáceres, C. Hernando, M. Mora, I.M. Pelayo, M.L. Puertas and C. Seara, Geodeticity of the contour of chordal graphs. *Discrete Appl. Math.* **156** (2008), no. 7, 1132–1142.

[2] G. Chartrand, D. Ervin, G.L. Johns and P. Zhang, Boundary vertices in graphs. *Discrete Math.* **263** (2003) 25–34.

[3] G. Chartrand, M. Schultz and S.J. Winters, On eccentric vertices in graphs. *Networks* **28** (1996) 181–186.

Intersecting families

Wednesday
11.40-12.00

Andrey Kupavskii

Ecole Polytechnique Fédérale de Lausanne, Switzerland

kupavskii@ya.ru

Put $[n] := \{1, 2, \ldots, n\}$ and let $2^{[n]}$ denote the power set of $[n]$. A subset $\mathcal{F} \subset 2^{[n]}$ is called a *family of subsets of* $[n]$, or simply a *family.* A family is called *intersecting*, if any two of its sets intersect.

The starting point of the research on intersecting families is the Erdős-Ko-Rado theorem, which states that any intersecting family of k-element subsets of $[n]$ contains at most $\binom{n-1}{k-1}$ elements, provided $n \geq 2k \geq 1$. The obvious example of an intersecting family attaining the bound is the family of all k-sets containing a fixed element.

A well-known stability result due to Hilton and Milner states that the largest intersecting family, which is not a subfamily of the Erdős-Ko-Rado family, consists of a k-set A, and all k-sets containing a fixed element $x \in [n] \setminus A$ and at least one element of A. A very strong result in this direction was obtained by Frankl [2], who gave a bound on the size of an intersecting family depending on the number of sets not containing the most popular element of the ground set. In a recent paper with D. Zakharov [4] we found a rather simple unified approach to such problems, reproving and strengthening many existing results, including the results of [2]. The proofs make use of perfect matchings in regular bipartite graphs.

The authors of [1] analyzed what is a *typical* intersecting family. They counted the total number $I(n, k)$ of intersecting families of k-subsets of $[n]$, and found out that for $n > 3k + 8\log k$ and $k \to \infty$ almost all intersecting families are *trivial*, that is, families in which all sets contain a fixed element. In [3] together with P. Frankl we improved their result. Roughly speaking, we proved that for $n \geq 2k + 2 + 2\sqrt{k \log k}$ and $k \to \infty$ almost all intersecting families are trivial, and almost all non-trivial intersecting families are the subfamilies of the Hilton-Milner families.

References:

[1] J. Balogh, SA. Das, M. Delcourt, H. Liu, M. Sharifzadeh, Intersecting families of dicrete structures are typically trivial, *J. Comb. Theory Ser. A* **132** (2015) 224–245.

[2] P. Frankl, Erdos-Ko-Rado theorem with conditions on the maximal degree, *J. Comb. Theory Ser. A* **46** (1987) N2, 252–263.

[3] P. Frankl, A. Kupavskii, Uniform s-cross-intersecting families, *Combinatorics, Probability and Computing* (2017).

[4] A. Kupavskii, D. Zakharov, Regular bipartite graphs and intersecting families, arXiv:1611.03129

Tuesday
12.00-12.20

Stanley Fiorini's work after edge-colouring

Josef Lauri

University of Malta, Malta

josef.lauri@um.edu.mt

After edge-colourings Stanley Fiorini turned his attention to the Graph Reconstruction Problem. Since he mainly concentrated on the reconstruction of planar graphs, this work had a distinctly topological flavour, with one of the main questions he first tackled being to determine when this statement is true: "A graph G is planar if and only if all its vertex-deleted subgraphs $G - v$ are planar." Reconstruction of planar graphs also required careful study of the notion of uniquely embeddable planar graphs. He and I showed that maximal planar graphs are reconstructible and we also considered the edge-reconstruction of graphs which triangulate surfaces, in particular the real projective plane.

Robin Wilson might have also influenced Stanley's decision to consider another area to which, some years later, he guided Irene Sciriha to turn her attention: spectral graph theory. Their first joint paper was on the generating functions of characteristic polynomials, focusing on the characterization of minimally non–outerplanar graphs, therefore also combining topological graph theory and the idea from reconstruction of a property which is or is not satisfied by any subgraph. Full attention soon shifted to singular graphs and the polynomial reconstruction problem. Together with Irene and Ivan Gutman, he studied trees with maximum nullity, contributing here interesting techniques from edge colouring.

With John Baptist Gauci he turned again to topological graph theory, first with a study of crossing numbers on generalised Petersen graphs. Then, Stanley and John, together with Tony Hilton and Keith Dugdale studied continuous k-to-1 functions on graphs, especially complete graphs. Here the idea is to look at graphs as topological spaces and to study when it is possible to define a function from a graph G onto a graph H in a continuous way such that each point of H has exactly k points of G mapped to it.

In these twenty minutes I shall attempt to summarise some of these results trying to point out some common themes running through them to give a flavour of Stanley's graph theoretic interests and achievements.

Cycles in sparse graphs

Thursday
15.20-15.40

Felix Lazebnik

University of Delaware, USA

fellaz@udel.edu

There are several sufficient conditions for a graph of order n to contain a cycle of length k, and, in particular, to be hamiltonian. Often these conditions do not hold in sparse graphs, i.e. in graphs of size $o(n^2)$, $n \to \infty$. In this talk we present several recent results on the existence of cycles of certain lengths (including hamiltonian cycles) in some families of sparse graphs.

References:

[1] J. Alexander, F. Lazebnik and A. Thomason, On the hamiltonicity of some sparse bipartite graphs, in preparation.

[2] S.M. Cioăba, F. Lazebnik and W. Li, On the spectrum of Wenger graphs, *J. Combin. Theory, Ser. B* **107** (2014) 132–139.

[3] M. Krivelevich and B. Sudakov, Sparse pseudo-random graphs are hamiltonian, *J. Graph Theory* **42** (2003) 17–33.

[4] F. Lazebnik, S. Sun and Y. Wang, Some families of graphs, hypergraphs and digraphs defined by systems of equations: a survey, *Lecture Notes of Seminario Interdisciplinare di Matematica* **14** (2017) 105–142.

[5] F. Lazebnik, A. Thomason and Y. Wang, On some cycles in Wenger graphs, to appear in *Acta Math. Appl. Sin. Engl.*

[6] K. Mellinger, F. Lazebnik and O. Vega, Embedding cycles in finite planes, *Electron. J. Comb.* **20** (2013) 1–17.

[7] G. E. Moorhouse, S. Sun and J. Williford, The eigenvalues of the graphs $D(4, q)$, *J. of Combin. Theory, Ser. B* **125** (2017) 1–20.

Thursday
16.00-16.20

Classifying bent functions by their Cayley graphs

Paul Leopardi

University of Melbourne, Australia
Australian Government - Bureau of Meteorology, Australia

paul.leopardi@gmail.com

Bent Boolean functions are fascinating and useful combinatorial objects, whose applications include coding theory and cryptography. The number of bent functions explodes with dimension, and various concepts of equivalence are used to classify them. In 1999 Bernasconi and Codenotti [1] noted that the Cayley graph of a bent function is strongly regular. This talk describes the concept of extended Cayley equivalence of bent functions, discusses some connections between bent functions, designs, and codes, and explores the relationship between extended Cayley equivalence and extended affine equivalence. SageMath scripts and SageMathCloud worksheets [2] are used to compute and display some of these relationships, for bent functions up to dimension 8.

References:

[1] A. Bernasconi and B. Codenotti, Spectral analysis of Boolean functions as a graph eigenvalue problem, *IEEE Transactions on Computers* **48(3)** (1999) 345–351.

[2] SageMath, Inc. *SageMathCloud Online Computational Mathematics*, (2016).

Tuesday
17.20-17.40

A family of largest-known degree 15 circulant graphs of arbitrary diameter

Robert Lewis

The Open University, UK

robert.lewis@open.ac.uk

The degree-diameter problem is the search for graphs with the largest possible number of vertices for a given degree and diameter. One of the simplest subcases is the restriction to circulant graphs, which are Cayley graphs of cyclic groups, but even this is a very difficult problem. Finding individual extremal graphs is a worthy pursuit, but a more satisfying challenge is to discover infinite families of extremal graphs where for a given degree the graphs are defined for every diameter.

In 1993 Chen and Jia [1] defined a simple construction for circulant graphs families of any even degree, which established a useful lower bound. Since then improved solutions have so far only been found up to degree 11. These families have generally been established by first finding extremal graphs of low diameter by means of exhaustive search with appropriate computer algorithms, and then fitting polynomials in the diameter to their order and generating sets. However this method is limited by the exponential complexity of the search space with increasing degree.

Using a new approach, families of largest-known circulant graphs of degrees 13, 14 and 15 have recently been discovered. Dougherty and Faber [2] proved the existence of certain infinite families of circulant graph families by establishing a relation with free Abelian groups and lattices, where the Cayley graph generators are mapped to lattice generator vectors. In the new approach the converse of this relation is used to determine the generating set of a circulant graph from a candidate matrix of lattice generator vectors. In the first application of this method, lattice generator matrices for largest-known degree 7 and 11 graphs were extrapolated to a degree 15 matrix which successfully generated a new family of largest-known degree 15 circulant graphs. In this talk I would like to describe the key steps in this new approach.

References:

[1] S. Chen and X.-D. Jia, Undirected loop networks, *Networks* **23** (1993) 257–260.

[2] R. Dougherty and V. Faber, The degree-diameter problem for several varieties of Cayley graphs, I: The Abelian case, *SIAM Journal on Discrete Mathematics* **17(3)** (2004) 478–519.

Enumeration of self-complementary circulant graphs of prime-power orders: old and new results

Friday 11.20-11.40

Valery Liskovets

National Academy of Sciences of Belarus, Belarus

liskov@im.bas-net.by

Let $SC(\delta;n)$ denote the number of self-complementary circulant graphs of order n, where $\delta = u$ or $\delta = d$ stands, respectively, for (ordinary) undirected or directed graphs. For $n = p^k$ ($p \geq 3$ prime, $k \geq 1$), the following formula is established:

$$SC(\delta;p^k) = A[\delta;p^k](-1),$$

where $A[\delta;n](x) = \sum_N A(\delta;n,N)x^N$ is the generating function for the numbers $A(\delta;n,N)$ of undirected or directed circulant graphs of order n with N edges. This formula goes back to [3], meets the general pattern proposed in [2] and solves completely the problem under consideration for $k = 1, 2$. Moreover, several formal identities are valid for the functions $SC(\delta;p)$ and $SC(\delta;p^2)$ including ones that involve related quantities such as the number of circulant tournaments. These identities are surveyed briefly. Then we consider in more detail the enumeration of self-complementary circulant graphs of orders 3^3 and 5^3 based on recent results [1]. In conclusion we discuss some relationships conjectured to be valid for $SC(\delta;p^3)$ in general.

References:

[1] V. Gatt, M. Klin, J. Lauri and V. Liskovets, Constructive and analytic enumeration of circulant graphs with p^3 vertices; $p=3,5$, *Preprint* arXiv:1512.07744.

[2] V. Liskovets, Some identities for enumerators of circulant graphs, *J. Algebraic Combin.* **18** (2003) 189–209.

[3] V. Liskovets and R. Pöschel, Counting circulant graphs of prime-power order by decomposing into orbit enumeration problems, *Discrete Math.* **214** (2000) 173–191.

Thursday
12.00-12.20

On the Incidence and Laplacian-like Energies of (bipartite) graphs

A. Dilek Maden

Selçuk University, Turkey

aysedilekmaden@selcuk.edu.tr

For a simple graph G and a real number α $(\neq 0, 1)$ the graph invariants s_α and σ_α are equal to the sum of powers of the non-zero signless Laplacian and Laplacian eigenvalues of G, respectively. Note that $s_{1/2}$ and $\sigma_{1/2}$ are equal to incidence and Laplacian-like energies of G, respectively. It is worth noting that $n\sigma_{-1}$ is also equal to the Kirchhoff index of G which has extensive applications in the theory of electric circuits, probabilistic theory and chemistry. Recently, the various properties and the estimates of these graph invariants (s_α and σ_α) have been well studied in the literature.

In this study, we present some generalized new bounds on s_α and σ_α of (bipartite) graphs. As a result of these bounds, we also obtain some generalized results on incidence and Laplacian-like energies of (bipartite) graphs.

Thursday
15.20-15.40

Vertex and edge transitive graphs over doubled cycles

Aleksander Malnič

University of Ljubljana and University of Primorska, Slovenia

aleksander.malnic@guest.arnes.si

(*joint work with* Boštjan Kuzman and Primož Potočnik)

We complete (and generalize) a result of A. Gardiner and C. Praeger on 4-valent symmetric graphs (European J. Combin, 15 (1994), 375–381). To this end we apply the lifting method in the context of elementary-abelian covering projections. In particular, for $p \neq 2$, vertex and edge transitive graphs whose quotient by some p-elementary abelian group of automorphisms is a cycle, are described in terms of cyclic codes.

Algorithmic and structural results on the existence of tropical subgraphs in vertex-colored graphs

Monday
17.40-18.00

Yannis Manoussakis

LRI, Univerity Paris South, France

yannis@lri.fr

In this work, we deal with tropical subgraphs in vertex-colored graphs. Vertex-colored graphs are useful in various situations. For instance, the Web graph may be considered as a vertex-colored graph where the color of a vertex represents the content of the corresponding page (red for mathematics, yellow for physics, etc.). Applications can also be found in bioinformatics (Multiple Sequence Alignment Pipeline or for multiple protein-protein Interaction networks).

Given a vertex-colored graph, a tropical subgraph (induced or otherwise) is defined to be a subgraph where each color of the initial graph appears at least once. Potentially, many graph invariants, such as the domination number, the vertex cover number, maximum matchings, independent sets, connected components, shortest paths, etc. can be studied in their tropical version. This notion is close to, but somewhat different from the colorful concept used for paths in vertex-colored graphs. It is also related to the concepts of color patterns or colorful used in bio-informatics.

Here, we study maximum tropical subgraphs and minimum tropical subgraphs in vertex-colored graphs. Some related work can be found in some other works, where the authors are looking for the minimum number of edges to delete in a graph such that all remaining connected components are colorful (i.e. do not contain two vertices of the same color). Note that in a tropical subgraph, adjacent vertices can receive the same color, thus a tropical subgraph may not be properly colored. Here we explain some results on our ongoing work on tropical dominating sets, vertex covers, connected subgraphs, maximum matchings and tropical homomorphisms.

Thursday
17.00-17.20

Intersection of transversals in the Latin square B_n, with applications to Latin trades

Trent G. Marbach

Monash University, Australia

trent.marbach@monash.edu

A paper by Cavenagh and Wanless diagnosed the possible intersection of any two transversals of B_n. We give a generalization of this problem for the intersection of μ transversals, and provide constructions and computational results for the cases where $\mu = 3, 4$. This result is then applied to the problem of finding μ-way k-homogeneous Latin trades, and along with a few new constructions, completes the spectrum of the existence 3-way k-homogeneous Latin trades for all but a small list of exceptions.

Friday
12.00-12.20

Parameterized complexity of metatheorems of fair deletion problems

Tomáš Masařík

Charles University,Czech Republic

masarik@kam.mff.cuni.cz

(*joint work with* Dušan Knop and Tomáš Toufar)

Deletion problems are those where given a graph G and a graph property π, the goal is to find a subset of vertices (or edges) such that after its removal the graph G will satisfy the property π. Typically, we want to minimize the number of elements removed. In fair deletion problems we change the objective: we minimize the maximum number of deletions in a neighborhood of a single vertex.

We study the parameterized complexity of metatheorems, where a graph property is expressed in a graph logic, of deletion problems with respect to several structural parameters of the graph.

The list of our results for the Vertex Deletion problem:

- The problem is W[1]-hard on tree-depth for any logic that can express the edgeless graph.
- The problem has an FPT algorithm for MSO_1 logic on graphs with bounded neighborhood diversity, or cluster vertex deletions, or twin cover.

The list of our results for the Edge Deletion problem:

- The problem is W[1]-hard on tree-depth for First order logic.

- The problem has an FPT algorithm for MSO_2 logic on graphs with bounded vertex cover.

References:

[1] D. Knop, T. Masařík, T. Toufar, Parameterized complexity of fair deletion problems II, *in preparation* (2017+).

[2] T. Masařík, T. Toufar, Parameterized complexity of fair deletion problems, *Accepted for publishing in the Proceedings of the Theory and Applications of Models of Computation 2017 (TAMC)* (2017).

Colourings of cubic graphs inducing isomorphic monochromatic subgraphs

Monday
11.40-12.00

Giuseppe Mazzuoccolo

University of Verona, Italy

giuseppe.mazzuoccolo@univr.it

(*joint work with* Marien Abreu, Jan Goedgebuer and Domenico Labbate)

At the beginning of the nineties, Ando conjectured that every cubic graph admits a (not necessarily proper) 2-colouring of the vertices such that the two induced monochromatic subgraphs are isomorphic. Similarly, Wormald conjectured, few years before, that every cubic graph of order a multiple of four admits a (not proper) 2-edge-colouring such that the two induced monochromatic subgraphs are isomorphic. Both conjectures are still largely open. Here, we present some new results on these conjectures. Moreover, we discuss the relation between them and another conjecture of Ban and Linial about the existence of a bisection of the vertices of a bridgeless cubic graph such that the two parts have all connected components of order at most two. In particular, we furnish some evidence to support these and some related conjectures. Moreover, we prove the Ban-Linial conjecture for all permutation snarks. Finally, we give a negative answer to a related question of Jackson and Wormald about certain decompositions of cubic graphs into linear forests.

Thursday
16.20-16.40

Analytic Combinatorics, Graph Transformations, and Stereochemistry

Daniel Merkle

University of Southern Denmark, Denmark

daniel@imada.sdu.dk

(*joint work with* Jakob L. Andersen, Christoph Flamm, Markus Nebel and Peter F. Stadler)

Counting chemical structures modelled as graphs dates back to the nineteenth century and since then this research field has been providing a fertile ground for interdisciplinary research in mathematics, computer science, and chemistry [1,2]. Graph transformation systems [3], where undirected graphs model molecules and double pushout (DPO) graph transformation rules model chemical reactions, are gaining importance, as large-scale chemical network analysis is becoming a prerequisite in understanding the possibilities in the universe of chemical compounds and reactions connecting them [4]. Analytic combinatorics [5] is a theory heavily used in many scientific disciplines, e.g. in computer science for the analysis of algorithms. In this contribution we will focus on combining the before-mentioned approaches, with the addition of modelling stereochemistry, i.e., the relative placement of atoms and their neighbours in space. Using analytic combinatorics we will analyze the combinatorial classes of alkanes and hydrocarbons, including stereochemical features. Multivariate generating functions will be presented that allow the inference of chemical properties. An extension to the DPO approach will allow for graph transformation systems for graphs with attributes that encode information about local geometry. The modelling approach is based on the "ordered list method", where an order is imposed on the set of incident edges of each vertex, and permutation groups determine equivalence classes of orderings that correspond to the same local spatial embedding. The DPO approach can be used to guide the combinatorial specifications used.

References:

[1] J.J. Sylvester, Chemistry and algebra, *Nature* **17** (1878) 284.

[2] G. Pólya and R.C. Read, *Combinatorial Enumeration of Groups Graphs and Chemical Compounds*, Springer, Berlin (1987).

[3] G. Rozenberg (Editor), *Handbook of Graph Grammars and Computing by Graph Transformation: Volume I. Foundations*, World Scientific Publishing Co., NJ, (1997).

[4] J.L. Andersen, C. Flamm, D. Merkle, and P.F. Stadler, *An Intermediate Level of Abstraction for Computational Systems*, Philosophical Transactions of the Royal Society A, to appear, (2017).

[5] P. Flajolét and R. Sedgewick, *Analytic Combinatorics*, Cambridge University Press, (2009).

On the Distinguishing Number of Cyclic Tournaments: Towards the Albertson-Collins Conjecture

Thursday
11.40-12.00

Kahina Meslem

USTHB University, Algeria

kmeslem@usthb.dz

(*joint work with* Eric Sopena)

A distinguishing r-labeling of a digraph G is a mapping λ from the set of vertices of G to the set of labels $\{1, ..., r\}$ such that no nontrivial automorphism of G preserves all the labels. The distinguishing number $D(G)$ of G is then the smallest r for which G admits a distinguishing r-labeling. Albertson and Collins [1] conjectured in 1999 that $D(T) = 2$ for every cyclic tournament T of (odd) order $2p + 1 \geq 3$, with $V(T) = \{0, ..., 2p\}$, and, more precisely, that the canonical 2-labeling λ^* given by $\lambda^*(i) = 1$ if and only if $i \leq p$ is distinguishing.

We prove that whenever one of the subtournaments of T induced by the vertices $\{0, ..., p\}$ or $\{p + 1, ..., 2p\}$ is rigid, T satisfies the Albertson-Collins Conjecture. Using this property, we prove that several classes of cyclic tournaments satisfy the Albertson-Collins Conjecture. Moreover, we also prove that every Paley tournament satisfies the Albertson-Collins Conjecture.

The full version of this paper is available at https://arxiv.org/pdf/1608.04866.pdf.

References:

[1] Michael O. Albertson and Karen L. Collins. A Note on Breaking the Symmetries of Tournaments, *Proc. 13th Southeastern Int. Conf. on Combinatorics, Graph Theory, and Computing. Congr. Numer.* **136** (1999) 129–131.

The Vertex Sign Balance of (Hyper)graphs

Friday
11.20-11.40

Dezső Miklós

Rényi Institute, Hungary

miklos.dezso@renyi.mta.hu

(*joint work with* Justin Ahmann, Elizabeth Collins-Wildman, John Wallace, Shun Yang, Yicong Guo and Gyula Y. Katona)

Pokrovskiy and, independently, Alon, Huang and Sudakov introduced the MMS (Manickam-Miklos-Singhi) property of hypergraphs: "for every assignment of weights to its vertices with nonnegative overall sum, the number of edges whose total weight is nonnegative is at least the minimum degree of H". This immediately leads to the definition of the following hypergraph parameter: The vertex sign balance of a hypergraph is the minimum number of edges whose total weight is nonnegative, where the minimum is taken over all assigments of weights to the vertices with nonnegative overall sum. The vertex sign balance is always between 0 and the minimum degree of the graph or hypergraph, both bounds being sharp. General and special properties (for graphs or three uniform hypergraphs) of this parameter will be presented. In particular, the characterization of the vertex sign balance of the graphs leads to the result that the question if a graph or hypergraph has the MMS property is NP-complete.

Monday
17.00-17.20

Distances between bicliques and structural properties of bicliques in graphs

Leandro Montero

Université de Luxembourg, Luxembourg

leandro.montero@uni.lu

(*joint work with* Marina Groshaus)

Let G be a simple graph. A *biclique* is a maximal bipartite complete induced subgraph of G. The *biclique graph* of a graph G, denoted by $KB(G)$, is the intersection graph of all bicliques of G. It was defined and characterized in [2]. However, no polynomial time algorithm is known for recognizing biclique graphs. Bicliques have applications in various fields, for example, biology [1], social networks [3], etc.

In this work, we first define the distance between bicliques as follows.
Definition. *Let G be a graph and let B, B' be bicliques of G. The distance between B and B' is defined as: $d(B,B') = min\{d(b,b') \ / \ b \in B,\ b' \in B'\}$.*

We give the following formula that relates the distance between bicliques in a graph G and the distance between their respectives vertices in $KB(G)$.
Lemma. *Let G be a graph and let B, B' be bicliques of G. Then $d_{KB(G)}(B,B') = \lfloor \frac{d_G(B,B')+1}{2} \rfloor + 1$.*

This is an important tool for proving some structural results on bicliques in graphs. Using this formula, we give a different (and easier) proof of the following necessity theorem for a graph to be a biclique graph.
Theorem ([2]) *Let $G = KB(H)$ for some graph H, then every induced P_3 of G is contained in an induced diamond or an induced gem of G.*

Moreover, using the distance formula we also prove that the condition of this theorem is not sufficient. Finally, we propose some conjectures about biclique graphs, for example the next one.
Conjecture. *Let $G = KB(H)$ for some graph H. Then G is Hamiltonian.*

References:

[1] D. Bu, Y. Zhao, L. Cai, H. Xue, X. Zhu, H. Lu, J. Zhang, S. Sun, L. Ling, N. Zhang, G. Li and R. Chen, Topological structure analysis of the protein-protein interaction network in budding yeast, *Nucleic Acids Research* **31** (2003) 2443–2450.

[2] M. Groshaus and J. L. Szwarcfiter, Biclique graphs and biclique matrices, *J. Graph Theory* **63** (2010) 1–16.

[3] R. Kumar, P. Raghavan, S. Rajagopalan and A. Tomkins, Trawling the web for emerging cyber-communities, *Proceeding of the 8th international conference on World Wide Web* (2000) 1481–1493.

On the Perron-Frobenius Theorem for reducible non-negative matrices

Tuesday
16.20-16.40

Joseph Muscat

University of Malta, Malta

joseph.muscat@um.edu.mt

The Perron-Frobenius theory is studied in the context of reducible non-negative matrices. It is shown that the support of a non-negative eigenvector of a non-negative matrix is a lower set of strong components of the digraph associated with the matrix. The Perron vector of a strong component has a unique positive extension downstream of it if, and only if, the other strong components in the downstream have strictly smaller Perron roots. As an application, these results allow for a simplified proof of Kirchhoff's theorem for the directed Laplacian of graphs.

Forbidden subposet problems with size restrictions

Thursday
12.00-12.20

Dániel T. Nagy

Eötvös Loránd University, Hungary

dani.t.nagy@gmail.com

Upper bounds to the size of a family of subsets of an n-element set that avoids certain configurations are proved. These forbidden configurations can be described by inclusion patterns and some sets having the same size. Our results are closely related to the forbidden subposet problems, where the avoided configurations are described solely by inclusions.

Monday
17.00-17.20

New bound on the size of saturating sets of projective planes

Zoltán Lóránt Nagy

Eötvös University, Hungary

nagyzoli@cs.elte.hu

Let Π_q be an arbitrary finite projective plane of order q. A subset S of its points is called *saturating* if any point outside S is collinear with a pair of points from S. The importance of such sets relies on connections to covering codes, algebraic curves over finite fields, sumset theory and complete arcs.

It is well known that $|S| > \sqrt{2q}+1$ for any saturating set S in Π_q. If q is a square and the plane is Desarguesian, then the existence of saturating sets with the same order of magnitude up to a constant factor is known due to Boros, Szőnyi and Tichler. [2]

However, in the general case when the plane is not necessarily Desarguesian or the order is arbitrary, we only have much weaker results. Following the footprints of Boros, Szőnyi and Tichler, Bartoli, Davydov, Giulietti, Marcugini and Pambianco obtained an estimate on the minimal size of a saturating set in Π_q.

Proposition [1] $\min |S| \leq (2 + o(1))\sqrt{q \ln q}$, if S is a saturating set in Π_q.

Our main theorem improves the constant term to $\sqrt{3}$.

Theorem [3] $\min |S| \leq (\sqrt{3} + o(1))\sqrt{q \ln q}$, if S is a saturating set in Π_q.

We present two proofs for this theorem. In the first one, we apply the probabilistic method. In the second one we proceed by showing that an advanced greedy-type algorithm also provides a saturating set of this size. Finally, we analyze the above approaches and point out their connection to hypergraph cover problems.

References:

[1] D. Bartoli, A.A. Davydov, M. Giulietti, M. S. Marcugini, F. Pambianco, Upper bounds on the smallest size of a saturating set in a projective plane and the Birthday problem, arXiv preprint (2015). arXiv:1505.01426

[2] T. Boros, T. Szőnyi, K. Tichler, On defining sets for projective planes. *Discrete Mathematics* **303(1)**, (2005) 17–31.

[3] Z.L. Nagy, Saturating sets in projective planes and hypergraph covers, arXiv preprint (2017). arXiv:1701.01379

Rhombus tilings of an even-sided polygon and projective quadrangulations

Tuesday
11.20-11.40

Atsuhiro Nakamoto

Yokohama National University, Japan

nakamoto@ynu.ac.jp

(*joint work with* Yuta Omizo and Yusuke Suzuki)

It is known that for any $k \geq 2$, the regular $2k$-gon P_{2k} with each segment of unit length admits a tiling by unit rhombi [1], for which the number of rhombi for tiling P_{2k} is $\binom{k}{2}$, and the number of congruence classes of the unit rhombi is $\lfloor \frac{k}{2} \rfloor$.

A *projective quadrangulation* Q is a map on the projective plane $\mathbb{P}$ with each face quadrilateral, and Q is *k-minimal* if every edge of Q is contained in a non-contractible cycle of length at least k, but for any face f of Q, the map obtained from Q by eliminating f by identifying two diagonal vertices of f no longer has a non-contractible cycle of length less than k. It is known that any two k-minimal projective quadrangulations can be transformed into each other by some local operation, called the Y-rotation [2].

In our talk, we first prove that the set of rhombus tilings of P_{2k} has a one-to-one correspondence with the set (Q, C), where Q is a k-minimal projective quadrangulation and C is a non-contractible k-cycle of Q.

Secondly, we apply this fact to find a new $Y\Delta$ equivalence class for maps on $\mathbb{P}$, in addition to the $Y\Delta$ equivalence class of minor-minimal k-representative maps on $\mathbb{P}$ found by Randby [3].

References:

[1] R. Kenyon, Tiling a polygon with parallelograms, *Algorithmica* **9** (1993) 382–397.

[2] A. Nakamoto and Y. Suzuki, Y-Rotations in k-minimal quadrangulations on the projective plane, *J. Graph Theory* **69** (2012) 301–313.

[3] S. Randby, Minimal embeddings in the projective plane, *J. Graph Theory* **25** (1997) 153–163.

Monday
17.20-17.40

The maximum size of a partial spread in a finite vector space

Esmeralda Năstase

Xavier University, U.S.A.

nastasee@xavier.edu

(*joint work with* Papa Sissokho)

Let $V(n,q)$ denote the vector space of dimension n over the finite field with q elements. A *partial t-spread* of $V(n,q)$ is a set of t-dimensional subspaces of $V(n,q)$ such that any two of them have trivial intersection. Let $r \equiv n \pmod t$. We prove that if $t > (q^r-1)/(q-1)$, then the maximum size, i.e., cardinality, of a partial t-spread of $V(n,q)$ is $(q^n - q^{t+r})/(q^t-1)+1$. This essentially settles a longstanding open problem in this area. Prior to this result, this maximum size was only known for $r=1$ and for $r=q=2$. In particular, this result also determines the clique number of the q-Kneser graph.

How to represent polyhedra, polyhedral tilings, and polytopes

Wednesday
11.40-12.00

Kengo Nishio

National Institute of Advanced Industrial Science & Technology (AIST), Japan

k-nishio@aist.go.jp

(*joint work with* Takehide Miyazaki)

Except for highly symmetric polytopes, there is no method for representing polytopes in general. This causes difficulty in understanding and describing complicated structures. For example, in materials science, the arrangements of atoms in liquids and glasses are often represented as polyhedral tilings. However, there has been no method for describing briefly what polyhedra are tiled in what way. To overcome this problem, we have created a theory for representing polytopes [1-3]. In this presentation, we will present the general outline of our theory.

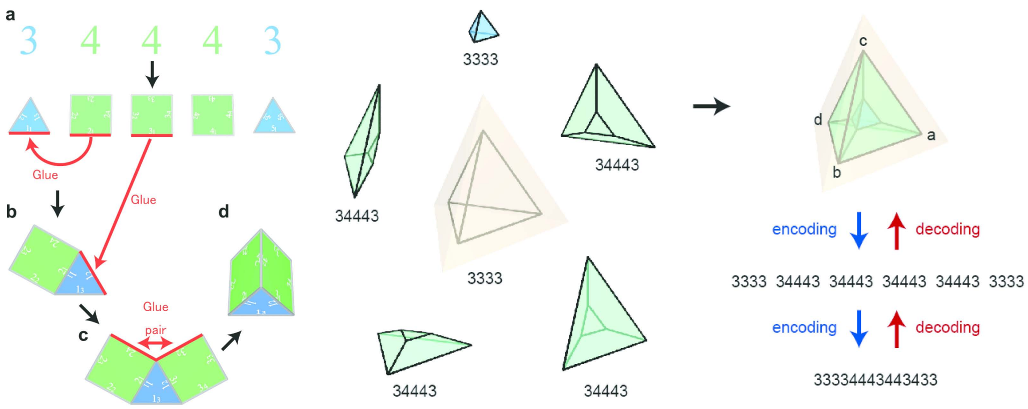


Figure 1: Overview of our theory.

References:

[1] K. Nishio and T. Miyazaki, How to describe disordered structures, *Scientific Reports* **6**, 23455 (2016).

[2] K. Nishio and T. Miyazaki, Creation of a mathematical method to express irregular atomic arrangements of amorphous materials – Discovery of a rule hidden in polyhedra, https://youtu.be/Z6qdZnJ8R-I (The subtitle is written in Japanese, but the movie can be understood without the subtitle.)

[3] K. Nishio and T. Miyazaki, Describing polyhedral tilings and higher dimensional polytopes by sequence of their two-dimensional components, *Scientific Reports* **7**, 40269 (2017).

Thursday
17.20-17.40

A new approach to catalog small graphs of high girth

Vivek S. Nittoor

Independent Consultant & Researcher, India
(formerly with the University of Tokyo, Japan)

vivek@nittoor.com

A catalog of a class of $(3, g)$ graphs for even girth g is introduced in this talk. A (k, g) graph is a graph with regular degree k and girth g. This catalog of $(3, g)$ graphs for even girth g satisfying $6 \leq g \leq 16$, has the following properties. Firstly, this catalog contains the smallest known $(3, g)$ graphs. An appropriate class of trivalent graphs for this catalog has been identified, such that the $(3, g)$ graph of minimum order within the class is also the smallest known $(3, g)$ graph. Secondly, this catalog contains $(3, g)$ graphs for more orders than other listings. Thirdly, the class of graphs have been defined so that a practical algorithm to generate graphs can be created. Fourthly, this catalog is infinite, since the results are extended into knowledge about infinitely many graphs.

The findings are as follows. Hamiltonian bipartite graphs have been identified as a promising class of trivalent graphs that can lead to a catalog of $(3, g)$ graphs for even girth g with graphs for more orders than other listings, that is also expected to contain a $(3, g)$ graph with minimum order. This catalog of $(3, g)$ graphs has many graphs outside of the vertex-transitive class. In order to make the computation more tractable, and at the same time, to enable deeper analysis on the results, symmetry factor has been introduced as a parameter that reflects the extent of rotational symmetry. The D3 chord index notation is introduced as a concise notation for trivalent Hamiltonian bipartite graphs. The D3 chord index notation is twice as compact as the LCF notation, which is known as a concise notation for trivalent Hamiltonian graphs. The D3 chord index notation can specify an infinte family of graphs. Results on the minimum order for existence of a $(3, g)$ Hamiltonian bipartite graph, and minimum value of symmetry factor for existence of a $(3, g)$ Hamiltonian bipartite graph are of wider interest from an extremal graph theory perspective.

Optimal Pebbling and Rubbling of Graphs with Given Diameter

Thursday
15.40-16.00

László F. Papp

Budapest University of Technology and Economics, Hungary

lazsa@cs.bme.hu

(*joint work with* Ervin Győri and Gyula Y. Katona)

A pebbling distribution P on graph G is a function mapping the vertex set to nonnegative integers. We can imagine that each vertex v has $P(v)$ pebbles. A pebbling move removes two pebbles from a vertex and places one at an adjacent one. A pebbling move is allowed if and only if the vertex loosing pebbles has at least two pebbles.

A vertex v is *reachable* under a distribution P, if there is a sequence of pebbling moves, such that each move is allowed under the distribution obtained by the application of the previous moves and after the last move v has at least one pebble. The *optimal pebbling number*, denoted by π_{opt}, is the smallest number m needed to guarantee a pebbling distribution of m pebbles from which any vertex is reachable. For a comprehensive list of references for the extensive literature see the book chapter [1].

Rubbling is a version of pebbling where an additional move is allowed. In this new move, one pebble each is removed at vertices v and w adjacent to a vertex u, and an extra pebble is added at vertex u. The *optimal rubbling number*, denoted by ρ_{opt}, is defined analogously to the optimal pebbling number.

It is known that if G is a connected graph, then $\pi_{\text{opt}}(G) \leq 2^{\text{diam}(G)}$ and this bound is sharp. We give a new short proof of this result. Besides, it proves that the same upper bound is sharp for the optimal rubbling number. A distance-k domination set S of a graph is a subset of the vertex set such that for each vertex v there is an element of S whose distance from v is at most k. The distance-k domination number of a graph, denoted by γ_k, is the size of the smallest distance-k domination set.

We prove that both $\pi_{\text{opt}}(G)$ and $\rho_{\text{opt}}(G)$ are at least $\min\left(2^k, \gamma_{k-1}(G)\right)$ for any positive integer k. Finally, we show that $\pi_{\text{opt}}(G) \geq \min(2^k, \gamma_{k-1}(G) + 2^{k-2}, \gamma_{k-2}(G))$ and $\rho_{\text{opt}}(G) \geq \min\left(2^k, \max\left(\frac{\gamma_{k-1}(G)}{2} + 2^{k-2}, \gamma_{k-1}(G)\right), \gamma_{k-2}(G)\right)$ where $k \geq 2$.

References:

[1] G. H. Hurlbert, Graph Pebbling, in: J. Gross, J. Yellen, P. Zhang, *Handbook of Graph Theory (Second Edition)*, Boca Raton (2014) 1428–1449.

Monday
12.20-12.40

Interval edge colorings of $(k^*, 2^*)$-bipartite graphs with bounded degree

Krzysztof Pastuszak

Gdańsk University of Technology, Poland

krzysztof_pastuszak@yahoo.com

(*joint work with* Anna Małafiejska, Michał Małafiejski and Krzysztof Ocetkiewicz)

An edge coloring of a graph G is an interval edge coloring if the set of colors on edges incident to every vertex of G forms an interval of integers. A bipartite graph G is called (α, β)-biregular if all vertices in one part of G have degree α and all vertices in the other part have degree β. A graph G is called a (α^*, β^*)-bipartite graph if G is a subgraph of a (α, β)-biregular graph and the maximum degree in one part is α and the maximum degree in the other part is β. The following problems of interval edge coloring are NP-complete: 6-coloring of $(6, 3)$-biregular graphs [1] and 5-coloring of $(5^*, 5^*)$-bipartite graphs [2]. In [3] the author showed that any $(3^*, 3^*)$-bipartite graph has an interval edge 4-coloring, which can be constructed in $O(n)$-time. In [2] the author proved that an interval edge k-coloring of every (k^*, k^*)-bipartite graph can be found in $O(n^{3/2})$-time (if it exists), for $k = 3, 4$. By results of [4], every $(2k, 2)$-biregular graph admits an interval edge $2k$-coloring, and every $(2k + 1, 2)$-biregular graph admits a $(2k + 2)$-coloring, for every $k \geq 1$.

In the paper we study the problem of interval edge colorings of $(k^*, 2^*)$-bipartite graphs, for $k = 3, 4$ and 5. We proved that every $(5^*, 2^*)$-bipartite graph admits an interval edge 6-coloring, which can be found in $O(n^{3/2})$-time, and we proved that an interval edge 5-coloring of a $(5^*, 2^*)$-bipartite graph can be found in $O(n^{3/2})$-time (if it exists). For $k = 4$, we showed that every $(4^*, 2^*)$-bipartite graph admits an interval edge 4-coloring, which can be found in $O(n)$-time. Moreover, we give the full characterisation of $(3^*, 2^*)$-bipartite graphs admitting interval edge 3-colorings.

References:

[1] A.S. Asratian, C.J. Casselgren, On interval edge colorings of (α, β)-biregular bipartite graphs, *Discrete Mathematics* **307** (2006) 1951–1956.

[2] K.Giaro, The complexity of consecutive Δ-coloring of bipartite graphs: 4 is easy, 5 is hard, *Ars Combinatoria* **47** (1997) 287–300.

[3] H.M. Hansen, Skemalægning med henblik på minimering af ventetid (in Danish), M.Sc. Thesis, University of Odense (1992).

[4] D. Hanson, C.O. Loten, B. Toft, On interval coloring of biregular bipartite graphs, *Ars Combinatoria* **50** (1998) 23–32.

Generalized forbidden subposet problems

Thursday
11.40-12.00

Balázs Patkós

Alfréd Rényi Institute of Mathematics, Hungarian Academy of Sciences, Hungary

patkos@renyi.hu

(*joint work with* Dániel Gerbner and Balázs Keszegh)

A subfamily $\{F_1, F_2, \ldots, F_{|P|}\} \subseteq \mathcal{F}$ of sets is a copy of a poset P in $\mathcal{F}$ if there exists a bijection $\phi : P \to \{F_1, F_2, \ldots, F_{|P|}\}$ such that whenever $x \leq_P x'$ holds, then so does $\phi(x) \subseteq \phi(x')$. For a family $\mathcal{F}$ of sets, let $c(P, \mathcal{F})$ denote the number of copies of P in $\mathcal{F}$, and we say that $\mathcal{F}$ is P-free if $c(P, \mathcal{F}) = 0$ holds. For any two posets P, Q let us denote by $La(n, P, Q)$ the maximum number of copies of Q over all P-free families $\mathcal{F} \subseteq 2^{[n]}$, i.e. $\max\{c(Q, \mathcal{F}) : \mathcal{F} \subseteq 2^{[n]}, c(P, \mathcal{F}) = 0\}$. This generalizes the well-studied parameter $La(n, P) = La(n, P, P_1)$ where P_1 is the one-element poset. In this talk we consider the problem of determining $La(n, P, Q)$ when P and Q are small posets, like chains, forks, the N poset, etc. Our main result determines $La(n, P_{h(Q)}, Q)$ up to some polynomial factor where Q is any complete multi-level poset, P_j is the chain of length j and $h(Q)$ is the height of Q (the length of the longest chain in Q). To obtain this, we solve (up to a polynomial factor) a problem on r-wise intersections in antichains.

Center, centroid and subtree core of trees

Thursday
17.20-17.40

Kamal Lochan Patra

National Institute of Science Education and Research, India

klpatra@niser.ac.in

(*joint work with* Dheer Noal Sunil Desai)

Let $T = (V, E)$ be a tree with vertex set V and edge set E. A vertex $v \in V$ is called a *central vertex* if the *eccentricity* $e(v)$ of v defined by $e(v) = \max\{d(u, v) : u \in V\}$ coincides with the *radius* $r(T)$ of T defined by $r(T) = \min\{e(v) : v \in V\}$. The *center* of T is the set of all central vertices of T. For $v \in V$, a *branch* (rooted) at v is a maximal subtree containing v as a pendant vertex. The *weight* $\omega(v)$ of v is the maximal number of edges in any branch at v. We say that v is a *centroid vertex* of T if $\omega(v) = \min_{u \in V} \omega(u)$. The *centroid* of T is the set of all centroid vertices of T. Let f be the function from V to the set of natural numbers $\mathbb{N}$ defined by $v \mapsto f(v)$, where $f(v)$ is the number of subtrees of T containing v. The *subtree core* of T is defined as the set of all vertices v for which $f(v)$ is maximum. It is known that each of the center, the centroid and the subtree core of T consists of either a single vertex or two adjacent vertices.

For $n \geq 5$ and $2 \leq g \leq n-3$, consider the tree $P_{n-g,g}$ on n vertices which is obtained by adding g pendant vertices to one of the end vertices of the path P_{n-g}. We call the trees $P_{n-g,g}$ as *path-star* trees. We prove that, among all trees on $n \geq 5$ vertices, the distance between the center and the subtree core, and the distance between the centroid and the subtree core both are maximized by some path-star trees. We then prove that the tree P_{n-g_0,g_0} maximizes both the distances among all path-star trees on n vertices, where g_0 is the smallest positive integer satisfying $2^{g_0} + g_0 > n - 1$.

Tuesday
17.40-18.00

Unified treatment of graphs and metric spaces

Jan Pavlík

Brno University of Technology, Czech Republic

pavlik@fme.vutbr.cz

We introduce a general approach to undirected graphs and metric spaces via generalization of metrics whose target set is a certain kind of an ordered monoid. Connected undirected graphs form one of several instances of an obtained structure. We will show how we can describe even disconnected graphs within this concept. Moreover, certain systems of neighborhoods on these spaces enable a definition of continuity. Depending on its choice, we define paths within the generalized metric spaces. Furthermore, we establish a concept of path-accessibility and use it for a description of classes of undirected graphs. Possibility of description of directed graphs will be discussed.

Monday
15.40-16.00

Inverses of Graphs

Soňa Pavlíková

Slovak University of Technology in Bratislava, Slovakia

sona.pavlikova@stuba.sk

(*joint work with* Daniel Ševčovič)

In this talk we study integrally invertible graphs. We investigate a class of the so-called positively and negatively invertible graphs. The positively invertible graphs are invertible graphs in the usual Godsil sense. Less attention has been given to the study of invertibility of non-bipartite graphs and their spectral properties. In the recent paper [4] we introduced a novel concept of negatively invertible graphs which can be considered as a natural extension to the concept of (positively) invertible graphs proposed by Godsil in [1] (see also [2,3]). The class of negatively invertible graphs, for which the inverse graph can be also constructed, contains important graphs arising from chemistry and other applications. We also propose a construction of integrally invertible graphs based on bridging two integrally invertible graphs over the set of their vertices. We derive sufficient conditions for

invertibility of bridged graphs. We furthermore analyze the spectral properties of these graphs and derive lower bounds for the least positive eigenvalue of the bridged graph. Finally, we present a complete list of graphs with a unique 1-factor on $m \leq 6$ vertices and determine their positive and negative invertibility.

References:

[1] C. D. Godsil, Inverses of Trees, *Combinatorica* **5** (1985) 33–39.

[2] S. Pavlíková and J. Krč-Jediný, On the inverse and dual index of a tree, *Linear and Multilinear Algebra*, **28** (1990) 93–109.

[3] S. Pavlíková, A note on inverses of labeled graphs, *Australasian Journal of Combinatorics*, **67** (2017) 222–234.

[4] S. Pavlíková and D. Ševčovič, On a Construction of Integrally Invertible Graphs and their Spectral Properties, *Linear Algebra and its Applications*, submitted.

Improving Upper Bounds for the Distinguishing Index

Monday
15.40-16.00

Monika Pilśniak

AGH University, Poland

pilsniak@agh.edu.pl

The distinguishing index of a graph G, denoted by $D'(G)$, is the least number of colours in an edge colouring of G not preserved by any non-trivial automorphism. We characterize all connected graphs G with $D'(G) \geq \Delta(G)$.

The definition of $D'(G)$ was introduced by Kalinowski and Pilśniak in [2]. They proved that if T is a tree of order at least 3, then $D'(T) \leq \Delta(T)$. Moreover, equality is achieved if and only if T is either a symmetric or a bisymmetric tree. We say that a tree is *symmetric* (or *bisymmetric*) if it has a central vertex v_0 (or a central edge e_0, respectively), all leaves are at the same distance from v_0 (or e_0) and all vertices that are not leaves have the same degree.

For finite connected graphs in general it was shown in [2] that $D'(G) \leq \Delta(G)$ unless G is C_3, C_4 or C_5. It follows for connected graphs that $D'(G) > \Delta(G)$ if and only if $D'(G) = \Delta(G) + 1$ and G is a cycle of length at most 5.

The equality $D'(G) = \Delta(G)$ holds for cycles of length at least 6, for K_4, $K_{3,3}$ and for all symmetric and bisymmetric trees. We show that $D'(G) < \Delta(G)$ for all other connected graphs.

This concept was also investigated for infinite graphs by Broere and Pilśniak in [1]. They proved that if G is a connected infinite graph such that the degree of every vertex is not greater than Δ, then $D'(G) \leq \Delta$. We present a quite recent improvement from [4]: if G is a connected infinite graph with maximum degree Δ, then

$D'(G) \leq \Delta - 1$ unless G is a double ray. Moreover we show that the bound given in this theorem is best possible for every finite $\Delta \geq 3$.

References:

[1] I. Broere and M. Pilśniak, The Distinguishing Index of Infinite Graphs, *Electron. J. Combin.* **23(1)** (2015) P1.78

[2] R. Kalinowski and M. Pilśniak, Distinguishing graphs by edge colourings, *European J. Combin.* **45** (2015) 124–131

[3] M. Pilśniak, Improving Upper Bounds for the Distinguishing Index, *Ars Math. Contemp.* accepted (2017)

[4] M. Pilśniak and M. Stawiski, Upper Bounds for the Distinguishing Index of Infinite Graphs, *manuscript*

Tuesday
12.20-12.40

Nonorientable Hypermaps of a given Type and Genus

Daniel Pinto

University of Coimbra, Portugal

dpinto@mat.uc.pt

Topologically, a hypermap is a cellular embedding of a connected hypergraph into a closed connected surface. If that underlying surface is orientable, we say that the hypermap is orientable. Otherwise, the hypermap is called nonorientable. We prove that given positive integers m, n with $2m^{-1} + n^{-1} < 1$ and an integer $g \geq 1$, there are infinitely many nonisomorphic compact nonorientable hypermaps of genus g and type (m, m, n), where m is the least common multiple of the valencies of the hypervertices and the least common multiple of the valencies of the hyperedges, and n is the least common multiple of the valencies of the hyperfaces. The technique we apply for the proof is based on the constructions used to demonstrate the same result for orientable hypermaps, making the suitable adjustments.

References:

[1] G. Jones and D. Pinto, Infinitely Many Hypermaps of Given Type and Genus, *Electronic Journal of Combinatorics* **17** (2010)

[2] D. Pinto, Nonorientable Hypermaps of a Given Type and Genus, *submitted*

Asymptotically optimal adjacent vertex distinguishing edge choice number

Monday
16.20-16.40

Jakub Przybyło

AGH University of Science and Technology, Poland

jakubprz@agh.edu.pl

(*joint work with* Jakub Kwaśny)

Let $G = (V, E)$ be a graph. Consider an edge colouring $c : E \to C$. For a given vertex $v \in V$, by $E(v)$ we denote the set of all edges incident with v in G, while the set of colours associated to these under c is denoted as:

$$S_c(v) = \{c(e) : e \in E(v)\}. \tag{1}$$

The colouring c is called *adjacent vertex distinguishing* if it is proper and $S_c(u) \neq S_c(v)$ for every edge $uv \in E$. It exists if and only if G contains no isolated edges. The least number of colours in C necessary to provide such a colouring is then denoted by $\chi'_a(G)$ and called the *adjacent vertex distinguishing edge chromatic number* of G. Obviously, $\chi'_a(G) \geq \chi'(G) \geq \Delta$, where Δ is the maximum degree of G, while it was conjectured [3] that $\chi'_a(G) \leq \Delta + 2$ for every connected graph G of order at least three and different from the cycle C_5. Hatami [1] proved the postulated upper bound up to an additive constant by showing that $\chi'_a(G) \leq \Delta + 300$ for every graph G with no isolated edges and with maximum degree $\Delta > 10^{20}$.

Suppose now that every edge $e \in E$ is endowed with a list of available colours L_e. The *adjacent vertex distinguishing edge choice number* of a graph G (without isolated edges) is defined as the least k so that for every set of lists of size k associated to the edges of G we are able to choose colours from the respective lists to obtain an adjacent vertex distinguishing edge colouring of G. We denote it by $\mathrm{ch}'_a(G)$. Analogously as above, $\mathrm{ch}'_a(G) \geq \mathrm{ch}'(G)$, while the best general result on the classical edge choosability implies that $\mathrm{ch}'(G) = (1 + o(1))\Delta$, see [2]. Generalizing this, a multistage probabilistic argument granting $\mathrm{ch}'_a(G) = (1 + o(1))\Delta$ for the class of all graphs without isolated edges shall be presented during my talk.

References:

[1] H. Hatami, $\Delta + 300$ is a bound on the adjacent vertex distinguishing edge chromatic number, *J. Combin. Theory Ser. B* **95** (2005) 246–256.

[2] M. Molloy, B. Reed, Near-optimal list colorings, *Random Structures Algorithms* **17** (2000) 376-402.

[3] Z. Zhang, L. Liu, J. Wang, Adjacent strong edge coloring of graphs, *Appl. Math. Lett.* **15** (2002) 623–626.

Monday
15.20-15.40

Cutting Planes for Union-Closed Families

Jonad Pulaj

Zuse Institute Berlin, Germany

pulaj@zib.de

Frankl's conjecture states that for any non-empty union-closed (UC) family there exists an element in at least half the sets of the family. We find previously unknown families $\mathcal{A}$ which contain an element that is frequent enough to satisfy Frankl's conjecture for all UC families $\mathcal{F} \supseteq \mathcal{A}$ using an algorithmic framework. Poonen's Theorem [2] characterizes the existence of weights which determine whether a given UC family $\mathcal{A}$ contains an element that satisfies Frankl's conjecture for all UC families $\mathcal{F} \supseteq \mathcal{A}$. We call such families $\mathcal{A}$ as above *Frankl-Complete* (FC). An UC family $\mathcal{A}$ is *Non–Frankl-Complete* (Non–FC), if and only if there exists an UC family $\mathcal{F} \supseteq \mathcal{A}$ such that each of the elements in $\mathcal{A}$ is in less than half the sets of $\mathcal{F}$. We design a cutting-plane method that computes the explicit weights which imply the existence conditions of Poonen's Theorem via exact rational integer programming. This method allows us to construct a counterexample to an eleven-year-old conjecture of Morris [1] about the structure of generators for Non–FC-families. Furthermore we answer in the negative two related questions of Vaughan [3] and Morris [1] regarding a simplified method for proving the existence of weights that yield FC-families.

References:

[1] R. Morris, FC-families, and improved bounds for Frankl's conjecture, *European Journal of Combinatorics* **23** (2006) 269–282.

[2] B. Poonen, Union-closed families, *J. Combin. Theory Ser. A* **59**(2) (1992) 253–268.

[3] T.P. Vaughan, A Note on the Union Closed Sets Conjecture, *J. Comb. Maths. and Comb. Comp.* **49** (2004) 73–84.

Nonlinear network dynamics under perturbations of the underlying graph

Thursday
16.00-16.20

Anca Rădulescu

State University of New York at New Paltz, U.S.A.

radulesa@newpaltz.edu

(*joint work with* Sergio Verduzco-Flores and Ariel Pignatelli)

Recent studies have been using graph theoretical approaches to model complex networks, and how hardwired circuitry relates to the ensemble's dynamic evolution in time. Understanding how configuration reflects on the coupled behavior in a system of dynamic nodes can be of great importance when investigating networks from the natural sciences. However, the effect of connectivity patterns on network dynamics is far from being fully understood.

We investigate the connections between edge configuration and dynamics in simple oriented networks with nonlinear nodes which update in both discrete and continuous time. In discrete time, we use complex quadratic nodes [1]. We define extensions of the traditional Julia and Mandelbrot sets, and we study the changes in their topology and fractal behavior in response to changes in the network's adjacencies. In continuous time, we illustrate coupled Wilson-Cowan equations [1,3]. We use configuration dependent phase spaces and a probabilistic extension of bifurcation diagrams in the parameter space, to investigate the relationship between classes of system architectures and classes of their possible dynamics.

In both cases, we differentiate between the effects on dynamics of altering edge weights, density, and configuration. We show that increasing the number of connections between nodes is not equivalent to strengthening a few connections, and that certain dynamic aspects are robust to the network configuration when the edge density is fixed. Finally, we interpret some of our results in the context of brain networks, synaptic restructuring and neural dynamics in learning networks.

References:

[1] A. Rădulescu, Neural network spectral robustness under perturbations of the underlying graph, *Neural Computation* **28** (2015).

[2] A. Rădulescu, A. Pignatelli, Real and complex behavior for networks of coupled logistic maps, *Nonlinear Dynamics* **84** (2016) 2025-2042.

[3] A. Rădulescu, S. Verduzco-Flores, Nonlinear network dynamics under perturbations of the underlying graph, *Chaos* **25** (2015).

Tuesday
17.00-17.20

Arc-Transitive Maps with underlying Rose Window Graphs

Alejandra Ramos Rivera

University of Primorska, Andrej Marušič Institute, Slovenia

alejandra.rivera@iam.upr.si

(*joint work with* Isabel Hubard, Primož Šparl)

A *map* $\mathcal{M}$ is an embedding of a connected graph Γ on a compact surface S without boundary, in such a way that $S \setminus \Gamma$ is a disjoint union of simply connected regions. A flag of a map $\mathcal{M}$ is defined as an incident triple $\{v, e, f\}$ of a vertex, an edge and a face of $\mathcal{M}$. The group of all automorphisms of a map $\mathcal{M}$ has a natural action on the set of its flags, $\mathcal{F}(\mathcal{M})$. The maps with most symmetries are maps having one orbit on $\mathcal{F}(\mathcal{M})$ (*reflexible maps*).

In this talk we focus on *arc-transitive maps*, that is maps for which their automorphism group is arc-transitive on the underlying graph. Of course, reflexible maps are examples of arc-transitive maps. As for maps $\mathcal{M}$, for which $\mathrm{Aut}(\mathcal{M})$ has two orbits on $\mathcal{F}(\mathcal{M})$, there are seven classes of such maps and only four of them result in arc-transitive maps; the corresponding classes are denoted by 2, 2_0, 2_1 and $2_{\{0,1\}}$.

The reflexible maps and maps in class 2 (*chiral maps*) have been extensively studied in the literature. Moreover, the maps in class 2_0 are related to the chiral ones via the Petrie operator (and their graphs and groups are the same). This leaves us with the maps of classes 2_1 and $2_{\{0,1\}}$ which are also related via the Petrie operator. It is known that the smallest admissible valency of the underlying graph of a map in class $2_{\{0,1\}}$ is four. It thus seems natural to first study maps of class $2_{\{0,1\}}$ with tetravalent underlying graphs.

In 2008 S. Wilson [2] introduced a family of tetravalent graphs now known as the Rose Window graphs and identified four families of arc-transitive members. The reflexible maps and maps in class 2 (and so in class 2_1) underlying these graphs were classified by I. Kovács, K. Kutnar and J. Ruff in [1]. In this talk we classify all maps in class $2_{\{0,1\}}$ (and so in class 2_0) underlying a Rose Window graph.

References:

[1] I. Kovács, K. Kutnar, J. Ruff, Rose window graphs underlying rotary maps, *Discrete Math.* **310** (2010) 1802–1811.

[2] S. Wilson, Rose Window Graphs, *Ars Math. Contemp.* **1** (2008) 7–19.

Toll number of the Cartesian and lexicographic product of graphs

Thursday
15.40-16.00

Polona Repolusk

University of Maribor, Slovenia

polona.repolusk@um.si

(*joint work with* Tanja Gologranc)

Toll convexity is a variation of the so-called interval convexity. A tolled walk T between u and v in G is a walk of the form $T : u, w_1, \ldots, w_k, v$, where $k \geq 1$, in which w_1 is the only neighbor of u in T and w_k is the only neighbor of v in T. As in geodesic or monophonic convexity, toll interval between $u, v \in V(G)$ is a set $T_G(u,v) = \{x \in V(G) : x \text{ lies on a tolled walk between } u \text{ and } v\}$. A set of vertices S is toll convex, if $T_G(u,v) \subseteq S$ for all $u, v \in S$. The toll closure $T_G[S]$ of a subset $S \subseteq V(G)$ is the union of toll intervals between all pairs of vertices from S, i.e. $T_G[S] = \cup_{u,v \in S} T_G(u,v)$. If $T_G[S] = V(G)$, we call S a toll set of a graph G. The order of a minimum toll set in G is called the toll number of G and is denoted by $tn(G)$.

First, the characterization of convex sets with respect to toll convexity in the Cartesian product of graphs will be reinvestigated. We will show that the toll number of the Cartesian product of two arbitrary graphs is 2. For the lexicographic product of graphs we will show that if H is not isomorphic to a complete graph, $tn(G \circ H) \leq 3 \cdot tn(G)$. Some necessary and sufficient conditions for $tn(G \circ H) = 3 \cdot tn(G)$ will be presented. Moreover, if G has at least two extreme vertices, a complete characterization will be presented. We also characterize graphs with $tn(G \circ H) = 2$ - this is the case iff G has an universal vertex and $tn(H) = 2$. Finally, the focus will be on the formula for $tn(G \circ H)$ - it can be described in terms of the so-called toll-dominating triples.

Indecomposable 1−factorizations of complete multigraphs

Thursday
11.40-12.00

Gloria Rinaldi

University of Modena and Reggio Emilia, Italy

gloria.rinaldi@unimore.it

(*joint work with* Simona Bonvicini)

A 1−factor of the complete multigraph λK_{2n}, $\lambda > 1$, is a spanning subgraph consisting of n edges that are pairwise independent. A 1−factorization of λK_{2n} is a partition of the edge set of λK_{2n} into 1−factors. The 1-factorization is said to be *indecomposable* if it cannot be represented as the union of 1-factorizations of $\lambda_0 K_{2n}$ and $(\lambda - \lambda_0) K_{2n}$, where $\lambda_0 < \lambda$. It is said to be *simple* if no 1-factor is repeated.

Despite the fact that 1−factorizations of λK_{2n} always exist (it is sufficient to take λ copies of a 1−factorization of K_{2n}, for example the classical 1−factorization of Lucas), the spectrum of values λ and n for which (simple or not simple) indecomposable 1−factorizations exist is still unknown. The problem of determining it seems hard in both cases (simple and not simple). Un upper bound for λ depending on n is presented in [1] and non existence results for few sporadic values of λ and n are given in [2]. Moreover, some existence results can be found in the literature, see for example [1], [2], [3].

We prove the existence for classes of parameters λ and n which were not previously considered. Precisely: we show that for every $n \geq 9$ and for every $(n-2)/3 \leq \lambda \leq 2n$ there exists an indecomposable and not simple 1-factorization of λK_{2n}. We can also exhibit some examples of indecomposable and not simple 1-factorizations for $n \in \{7, 8\}$, $(n-2)/3 \leq \lambda \leq n$, and for $n \in \{5, 6\}$, $(n-2)/3 \leq \lambda \leq n-2$. We use these constructions to prove the existence of simple and indecomposable 1-factorizations of λK_{2n} for every $n \geq 18$ and for every $2 \leq \lambda \leq 2\lfloor n/2 \rfloor - 1$. In [2] a simple and indecomposable 1−factorization of λK_{p+1}, with p an odd prime, and $\lambda = \frac{p-1}{2}$ is presented. Using structural properties of finite Galois Fields, we generalize this result to λK_{p^m+1} with $\lambda = \frac{p^m-1}{2}$ and $m \geq 1$.

References:

[1] A.H. Baartmans and W.D. Wallis, Indecomposable factorizations of multigraphs, *Discrete Math.* **78** (1989) 37–43.

[2] C.J. Colbourn, M.J. Colbourn, and A. Rosa, Indecomposable 1-factorizations of the complete multigraph, *J. Austral. Math. Soc. Ser A* **39** (1985) 334–343.

[3] C. Wensong, New constructions of simple and indecomposable 1-factorizations of complete multigraphs, *J Stat Plann Inference* **94** (2001) 181–196.

Monday
17.40-18.00

The cardinality of the lattice of characteristic subspaces

Alicia Roca

Universitat Politècnica de València, Spain

aroca@mat.upv.es

(*joint work with* David Mingueza and M. Eulàlia Montoro)

Given $J \in M_n(\mathbb{F})$ a nilpotent Jordan matrix and $\mathbb{F}$ a field, a J-invariant subspace is called characteristic (respectively hyperinvariant) if it is also T-invariant for all of the nonsingular matrices T (respectively, matrices T) commuting with J. We denote by $Chinv(J)$ and $Hinv(J)$ the lattices of characteristic and hyperinvariant subspaces, respectively. Obviously:

$$Hinv(J) \subseteq Chinv(J).$$

It is known that $Chinv(J) = Hinv(J)$ if $\mathbb{F} \neq GF(2)$. We understand

$$Chinv(J) = Hinv(J) \cup (Chinv(J) \setminus Hinv(J)).$$

The cardinality of $Hinv(J)$ is known. For $\mathbb{F} = GF(2)$, the subspaces in $Chinv(J) \setminus Hinv(J)$ are characterized as direct sums $Z \oplus Y$, where Y, Z are certain types of subspaces associated to a so-called char-tuple.

We compute the number of characteristic non-hyperinvariant subspaces over the field $GF(2)$. Results are highly combinatorial. We give a recurrent formula to compute the cardinality of subspaces of the type Z, and give an algorithm to calculate the number of subspaces of the type Y. This algorithm leads to the construction of a matrix which generalizes the Pascal matrix.

Quasi-symmetric 2-$(64, 24, 46)$ designs derived from $AG(3, 4)$

Tuesday
11.20-11.40

Sanja Rukavina

University of Rijeka, Croatia

sanjar@math.uniri.hr

(*joint work with* Dean Crnković, B. G. Rodrigues and Vladimir D. Tonchev)

In this talk we present the enumeration of quasi-symmetric 2-$(64, 24, 46)$ designs supported by the dual code $C^{\perp}$ of the binary linear code C spanned by the lines of $AG(3, 4)$. It is shown that $C^{\perp}$ supports exactly 30,264 nonisomorphic quasi-symmetric 2-$(64, 24, 46)$ designs.

The block graph of a quasi-symmetric 2-$(64, 24, 46)$ design with block intersection numbers 8 and 12, where two blocks are adjacent if they share 12 points, is a strongly regular graph with parameters $(336, 80, 28, 16)$. The block graphs of the 2699 nonisomorphic quasi-symmetric 2-$(64, 24, 46)$ designs admitting an automorphism of order 128, split into 2371 isomorphism classes of strongly regular graphs with parameters $(336, 80, 28, 16)$. The automorphism groups of those strongly regular graphs are computed.

Vertex connectivity of the power graph of a finite cyclic group

Tuesday
17.20-17.40

Binod Kumar Sahoo

National Institute of Science Education and Research, India

bksahoo@niser.ac.in

(*joint work with* Sriparna Chattopadhyay and Kamal Lochan Patra)

The *power graph* $\mathcal{P}(G)$ of a group G is the simple undirected graph with vertex set G, in which two distinct vertices are adjacent if and only if one of them can be obtained as an integral power of the other. Here we consider the finite cyclic group C_n of order n and study the vertex connectivity $\kappa(\mathcal{P}(C_n))$ of $\mathcal{P}(C_n)$. Recall that $\kappa(\mathcal{P}(C_n))$ is the minimum number of vertices which need to be removed from C_n so that the induced subgraph of $\mathcal{P}(C_n)$ on the remaining vertices is disconnected or has only one vertex. If $n = 1$, then $\kappa(\mathcal{P}(C_1)) = 0$. If $n = p^m$ for some prime p and positive integer m, then $\mathcal{P}(C_{p^m})$ is a complete graph and so $\kappa(\mathcal{P}(C_{p^m})) = p^m - 1$. Therefore, we assume that n is divisible by at least two distinct primes. Let $n = p_1^{n_1} p_2^{n_2} \cdots p_r^{n_r}$, where $r \geq 2$, $n_1, n_2, \cdots, n_r$ are positive integers and $p_1, p_2, \cdots, p_r$ are distinct primes with $p_1 < p_2 < \cdots < p_r$. Let $\phi : \mathbb{N} \longrightarrow \mathbb{N}$ denote the Euler's totient function. For a given subset X of C_n, we define $\overline{X} = C_n \setminus X$ and denote by $\mathcal{P}(\overline{X})$ the induced subgraph of $\mathcal{P}(C_n)$ with vertex set $\overline{X}$. We prove the following hold:

(i) If $2\phi(p_1 \cdots p_{r-1}) > p_1 \cdots p_{r-1}$, then

$$\kappa(\mathcal{P}(C_n)) = \phi(n) + p_1^{n_1-1} \cdots p_{r-1}^{n_{r-1}-1} p_r^{n_r-1} \left[p_1 p_2 \cdots p_{r-1} - \phi(p_1 p_2 \cdots p_{r-1})\right].$$

There is only one subset X of C_n with $|X| = \kappa(\mathcal{P}(C_n))$ and $\mathcal{P}(\overline{X})$ disconnected.

(ii) If $2\phi(p_1 \cdots p_{r-1}) < p_1 \cdots p_{r-1}$, then

$$\kappa(\mathcal{P}(C_n)) \leq \phi(n) + p_1^{n_1-1} \cdots p_{r-1}^{n_{r-1}-1} \left[p_1 \cdots p_{r-1} + \phi(p_1 \cdots p_{r-1})(p_r^{n_r-1} - 2)\right].$$

(iii) If $2\phi(p_1 \cdots p_{r-1}) = p_1 \cdots p_{r-1}$, then $r = 2$, $p_1 = 2$ (so that $n = 2^{n_1} p_2^{n_2}$) and

$$\kappa(\mathcal{P}(C_n)) = \phi(n) + 2^{n_1-1} p_2^{n_2-1}.$$

There are n_2 subsets X of C_n with $|X| = \kappa(\mathcal{P}(C_n))$ and $\mathcal{P}(\overline{X})$ disconnected.

In order to show that the bound in (ii) is sharp for certain values of n, we prove that if $r = 3$ and $2\phi(p_1 p_2) < p_1 p_2$, then $p_1 = 2$ and

$$\kappa(\mathcal{P}(C_n)) = \phi(n) + 2^{n_1-1} p_2^{n_2-1} \left[(p_2 - 1) p_3^{n_3-1} + 2\right].$$

Further, there is only one subset X of C_n with $|X| = \kappa(\mathcal{P}(C_n))$ such that $\mathcal{P}(\overline{X})$ is disconnected.

Blocking sets of certain line sets to a hyperbolic quadric in $PG(3, q)$, q even

Tuesday
12.20-12.40

Bikramaditya Sahu

National Institute of Science Education and Research, India

bikram.sahu@niser.ac.in

Let $q = 2^t, t \geq 1$, and $PG(3, q)$ be the 3-dimensional projective space over a finite field of order q. We denote by $\mathcal{P}$ and $\mathcal{L}$ the point set and the line set of $PG(3, q)$, respectively. For a subset L of $\mathcal{L}$, an L-blocking set of $PG(3, q)$ is a subset B of $\mathcal{P}$ such that every line in L contains at least one point of B. Let $\mathcal{H}$ be a hyperbolic quadric in $PG(3, q)$, that is, a non-degenerate quadric of Witt index two. Let E (respectively; T, S) denote the set of all lines of $PG(3, q)$ which are external (respectively; tangent, secant) to $\mathcal{H}$. If $L = E \cup T \cup S$ and B is an L-blocking set of $PG(3, q)$, then $|B| \geq \frac{q^3-1}{q-1}$ and equality holds if and only if B is a plane of $PG(3, q)$ [2]. In this talk, we shall address the following combinatorial question: For $L \in \{E, T, S, E \cup T, E \cup S, T \cup S\}$, find the minimum size of an L-blocking set and describe all L-blocking sets of that cardinality. We note that this question was answered in [1] for $L = E$ and in [4] for $L = E \cup S$.

The point-line geometry $W(q)$ with point set $\mathcal{P}$ and line set consisting of the totally isotropic lines of $PG(3, q)$ with respect to a symplectic polarity is a generalized quadrangle of order q [3]. An ovoid of $W(q)$ is a set O of points with the property that each line of $W(q)$ meets O in one point. The known ovoids of $PG(3, q)$ are of two types: (i) the elliptic quadrics which exist for all $t \geq 1$, (ii) the Tits ovoids which exist for odd $t \geq 3$. However, classifying all ovoids of $W(q)$ is still an open problem. Let Γ be the graph whose vertices are the ovoids of $W(q)$, in which two distinct vertices are adjacent if they intersect at one point. Though all vertices of Γ are not known, the authors in [4] gave a bound on the clique number of Γ as an application of their result on $(E \cup S)$-blocking sets connecting combinatorics and graph theory. We shall discuss this bound as well.

References:

[1] P. Biondi and P. M. Lo Re, On blocking sets of external lines to a hyperbolic quadric in $PG(3, q)$, q even, *J. Geometry* **92** (2009) 23–27.

[2] R. C. Bose and R. C. Burton, A characterization of flat spaces in a finite geometry and the uniqueness of the Hamming and the MacDonald codes, *J. Combinatorial Theory* **1** (1966) 96–104.

[3] S. E. Payne and J. A. Thas, Finite Generalized Quadrangles, EMS series of lectures in Mathematics, European Mathematical Society, Zurich, 2009.

[4] B. K. Sahoo and N. S. N. Sastry, Binary codes of the symplectic generalized quadrangle of even order, *Des. Codes Cryptogr.* **79** (2016) 163–170.

On the Spouse-Loving Variant of the Oberwolfach Problem

Thursday
12.20-12.40

Mateja Šajna

University of Ottawa, Canada

msajna@uottawa.ca

(*joint work with* Noah Bolohan, Iona Buchanan and Andrea Burgess)

The well-known *Oberwolfach Problem* asks the following: Given t round tables of sizes $m_1, \ldots, m_t$ such that $m_1+\ldots+m_t = n$, is it possible to seat n people around the t tables for an appropriate number of meals so that every person sits next to every other person exactly once? In graph-theoretic terms, the question is asking whether K_n can be decomposed into 2-factors, each a vertex-disjoint union of cycles of lengths $m_1, \ldots, m_t$, whenever $m_1 + \ldots + m_t = n$. Since a graph with odd-degree vertices cannot admit a 2-factorization, Huang et al. proposed the analogous problem for $K_n - I$, the complete graph of even order with a 1-factor removed. They called it the *spouse-avoiding variant* since it models a sitting arrangement of $\frac{n}{2}$ couples, where each person gets to sit next to each other person, except their spouse, exactly once.

For both of these two basic variants of the Oberwolfach Problem, the cases with uniform cycle length were completely solved decades ago [Alspach and Häggkvist, Alspach et al., Hoffman and Schellenberg]. In addition, many solutions are now known for variable cycle lengths; most notably, the problem is solved for $m_1, \ldots, m_t$ all even [Bryant and Danziger]; for $t = 2$ [Traetta]; and for $n \leq 40$ [Deza et al.]. However, in general, it is still wide open.

The spouse-avoiding variant of the Oberwolfach Problem can also be viewed as the *maximum packing* variant. This talk, however, pertains to the *minimum covering* version of the problem; in other words, we are interested in decomposing $K_n + I$ (the complete graph of even order with a 1-factor duplicated) into 2-factors, each a vertex-disjoint union of cycles of lengths $m_1, \ldots, m_t$, where $m_1 + \ldots + m_t = n$. We denote this problem by $\mathrm{OP}^+(m_1, \ldots, m_t)$, or $\mathrm{OP}^+(n; m)$ when $m_1 = \ldots = m_t = m$. This variant, nicknamed the *spouse-loving variant*, models a situation where it is preferable for each person to sit next to exactly one other person twice (instead of never), and next to every other person exactly once.

Under the disguise of *resolvable minimum coverings by triples*, $\mathrm{OP}^+(n; 3)$ has been shown to have a solution whenever $3|n$ and $n \geq 18$ [Assaf et al., Lamken and Mills], and from existing solutions to the spouse-avoiding variant, it follows easily that $\mathrm{OP}^+(m_1, \ldots, m_t)$ has a solution whenever $m_1, \ldots, m_t$ are all even. In this talk, we show that for odd $m \geq 5$, $\mathrm{OP}^+(n; m)$ has a solution whenever $m|n$, except possibly for $n = 4m$.

Forbidden Pairs of Minimal Quadratic and Cubic Configurations

Monday
12.00-12.20

Attila Sali

Alfréd Rényi Institute of Mathematics, Hungarian Academy of Sciences, Hungary

sali.attila@renyi.mta.hu

(*joint work with* Sam Spiro)

A matrix is *simple* if it is a (0,1)-matrix and there are no repeated columns. Given a (0,1)-matrix F, we say a matrix A has F as a *configuration*, denoted $F \prec A$, if there is a submatrix of A which is a row and column permutation of F. Let $|A|$ denote the number of columns of A. A simple (0,1)-matrix A can be considered as vertex-edge incidence matrix of a hypergraph without repeated edges. A configuration is a trace of a subhypergraph of this hypergraph. Let $\mathcal{F}$ be a family of matrices. We define the extremal function $\text{forb}(m, \mathcal{F}) = \max\{|A| \colon A \text{ is an } m-\text{rowed simple matrix and has no configuration } F \in \mathcal{F}\}$. We consider pairs $\mathcal{F} = \{F_1, F_2\}$ such that F_1 and F_2 have no common extremal construction and derive that individually each $\text{forb}(m, F_i)$ has greater asymptotic growth than $\text{forb}(m, \mathcal{F})$, extending research started by Anstee and Koch. They determined $\text{forb}(m, \{F, G\})$ for all pairs $\{F, G\}$, where both members are *minimal quadratics*, that is both $\text{forb}(m, F) = \Theta(m^2)$ and $\text{forb}(m, G) = \Theta(m^2)$, but no proper subconfiguration of F or G is quadratic. We take this one step further. That is, we consider cases when one of F or G is a simple minimal cubic configuration and the other one is a minimal quadratic or minimal simple cubic. We solve all cases when the minimal simple cubic configuration has four rows. If a conjecture of Anstee is true, then there is no minimal simple cubic configuration on 5 rows. About the six-rowed ones we observe that $\text{forb}(m, \{F, G\})$ is quadratic if F is minimal quadratic and G is a 6-rowed minimal cubic in all, but one cases. In the remaining case we believe that non-existence of common quadratic product construction indicates that the order of magnitude is $o(m^2)$.

Dominating sets in Circulant graphs

Monday
16.40-17.00

Tamene K. Samuel

University of Ngaoundere, Cameroon

samueltken@gmail.com

A Cayley graph on a group Γ is a graph G with the elements of Γ forming the vertex–set and edge–set defined by a generating set $S \subset \Gamma$ such that vertex g is adjacent to vertex h if and only if $h = gs$ for some $s \in S$. A circulant graph is a Cayley graph on a finite cyclic group $\mathbb{Z}_n$ denoted by $Circ(n, S) = Cay(\mathbb{Z}_n, S)$. In this paper, we address the *Dominating Set* and *Independent Dominating Set* problems in circulant graphs. We focus our attention to special generating sets consisting of consecutive integers, $S = \{a, a+1, ...a+k-1, n-a, ..., n-(a+k-1)\}$ where a, k and n are integers such that $1 \leq a \leq k+2$ and $a+k \leq \frac{n+1}{2}$. We give the domination number and the independent domination number, and describe some of these sets for such circulant graphs. We also give a general condition for a subgroup to be an independent dominating set of a circulant graph.

Thursday
17.40-18.00

On the fractional metric dimension of Mycielski graphs

Suhadi Wido Saputro

Bandung Institute of Technology, Indonesia

suhadi@math.itb.ac.id

(*joint work with* Tia Qibtiatul Munawaroh)

Let G be a simple connected graph. A vertex z in G *resolves* two vertices u and v in G if the distance from u to z is not equal to the distance from v to z. A set of vertices $R_G\{u, v\}$ is a set of all resolving vertices of u and v in G. For every two distinct vertices u and v in G, a *resolving function* f of G is a real function $f : V(G) \to [0, 1]$ such that $f(R_G\{u, v\}) = \sum_{z \in R_G\{u,v\}} f(z)$ is at least 1. The minimum value of $|f| = \sum_{z \in V(G)} f(z)$ from all resolving functions f of G is called the *fractional metric dimension* of G. In this paper, we consider Mycielski graphs. For a connected graph G of order n with $V(G) = \{v_1, v_2, \ldots, v_n\}$, a *Mycielski graph* of G, denoted by $\mu(G)$, is a graph with $V(\mu(G)) = \{x_i, y_i \mid 1 \leq i \leq n\} \cup \{z\}$ and $E(\mu(G)) = \{x_i x_j, x_i y_j \mid v_i v_j \in E(G)\} \cup \{y_i z \mid 1 \leq i \leq n\}$. We give sharp lower and upper bounds for the fractional metric dimension of Mycielski graphs. We also determine an exact value of the fractional metric dimension of $\mu(G)$ where G is a complete graph, a star graph, or a cycle graph.

Thursday
12.20-12.40

Explicit constructions of Ramanujan graphs

Shohei Satake

Kobe University, Japan

sheq3141@gmail.com

Ramanujan graphs were introduced by Lubotzky-Phillips-Sarnak [3]. Let G be a k-regular graph and $\lambda(G) = \max\{|\lambda| \mid \lambda \in \mathrm{Spec}(G), |\lambda| \neq k\}$. Here, $\mathrm{Spec}(G)$ is the set of all eigenvalues of the adjacency matrix of G. A k-regular graph G is a Ramanujan graph if $\lambda(G) \leq 2\sqrt{k-1}$. Ramanujan graphs connect to various areas of mathematics, such as number theory and group theory, and have wide applications in coding theory, computer science and so on. To give explicit constructions of Ramanujan graphs is a very interesting and meaningful problem. In these two decades, many explicit constructions of families of Ramanujan graphs were given. Our main purpose is to give explicit constructions of infinite families of Ramanujan graphs (with unbounded degree). In particular, we focus on Cayley graphs and Cayley sum graphs. As remarked by Chung [1] and Li et.al. [2], the eigenvalues of such graphs can be expressed by character sums. In this talk, we construct Cayley graphs and Cayley sum graphs which are Ramanujan over finite fields $\mathbb{F}_q$, residue rings $\mathbb{Z}_{p^e}$ with prime power order and Galois rings $R_{p^2,r}$.

References:

[1] F. R. K. Chung, Diameters and eigenvalues, *J. Amer. Math. Soc.* **2** (1989) no. 2, 187–196.

[2] W.-C. W. Li, K. Feng, Character sums and abelian Ramanujan graphs, *J. Number Theory* **41** (1992) no. 2, 199–217.

[3] A. Lubotzky, R. Phillips, P. Sarnak, Ramanujan graphs, *Combinatorica* **8** (1988) no. 3, 261–277.

Boundary Enumerator Polynomial of Hypercubes in Fibonacci Cubes

Thursday
17.40-18.00

Elif Saygı

Hacettepe University, Turkey

esaygi@hacettepe.edu.tr

(*joint work with* Ömer Eğecioğlu)

Let $\mathbb{D}_{n,k}(d)$ be the polynomial that enumerates the boundary of the k-dimensional hypercubes Q_k in the Fibonacci cube Γ_n of dimension n, where the degree of any monomial in $\mathbb{D}_{n,k}(d)$ is the number of edges in the boundary of the corresponding Q_k and the coefficient of that monomial shows the number of such hypercubes. In this work, we obtain recursive relations for $\mathbb{D}_{n,k}(d)$ and its generating function by using the fundamental decomposition of Γ_n. Our motivation is the wealth of known results on the degree distribution of the vertices in Γ_n, which corresponds to the special case $k = 0$ in our formulation. In [2] it is shown that the degrees of the vertices in Γ_n are between $\left\lfloor \frac{n+2}{3} \right\rfloor$ and n. Furthermore, in [1] depending on the recursive structure of Γ_n, a recursive formula for computing the degree of any vertex is given. In [4] vertices of degrees n, $n-1$, $n-2$ and $n-3$ are explicitly described and the degrees of vertices are used to investigate the domination number Γ_n. Finally in [3], by deriving and solving a corresponding system of linear recurrences, the number of vertices of any degree in Γ_n is presented and a direct approach to this problem by considering degrees via the partition of $V(\Gamma_n)$ into strings of any weight is given. In the last section of [3] a method using generating functions is also presented. In this work we extend this approach to find $\mathbb{D}_{n,k}(d)$. As noted in [3] this method is more involved and complicated as we are considering arbitrary k-dimensional hypercubes instead of vertices. The recursive relation obtained for the number of vertices in Γ_n of degree m in [3, Section 6] turns out to be contained in the specialization of our formulation as $\mathbb{D}_{n,0}(d) = d\,(\mathbb{D}_{n-1,0}(d) + \mathbb{D}_{n-2,0}(d) + \mathbb{D}_{n-3,0}(d)) - d^2\mathbb{D}_{n-3,0}(d)$.

References:

[1] J. A. Ellis-Monaghan, D. A. Pike and Y. Zou, Decycling of Fibonacci cubes, *Australas. J. Combin.* **35** (2006) 31–40.

[2] W.-J. Hsu, Fibonacci cubes–a new interconnection technology. *IEEE Trans. Parallel Distrib. Syst.* **4(1)** (1993) 3–12.

[3] S. Klavžar, M. Mollard and M. Petkovšek, The degree sequence of Fibonacci and Lucas cubes, *Discrete Math.* **311(14)** (2011) 1310–1322.

[4] D. A. Pike and Y. Zou, The domination number of Fibonacci cubes, *J. Combin. Math. Combin. Comput.* **80** (2012) 433–444.

Tuesday
16.20-16.40

Maximum independent sets near the upper bound

Ingo Schiermeyer

Technische Universität Bergakademie Freiberg, Germany

Ingo.Schiermeyer@tu-freiberg.de

The size of a largest independent set of vertices in a given graph G is denoted by $\alpha(G)$ and is called its *independence number* (or *stability number*). Given a graph G and an integer K, it is NP-complete to decide whether $\alpha(G) \geq K$. An upper bound for the independence number $\alpha(G)$ of a given graph G with n vertices and m edges is given by
$\alpha(G) \leq p := \lfloor \frac{1}{2} + \sqrt{\frac{1}{4} + n^2 - n - 2m} \rfloor$ ([1]).

In this talk we will present the following results.

Theorem There exists an algorithm with time complexity $O(n^2)$ that, given as an input a graph G with n vertices, m edges, $p := \lfloor \frac{1}{2} + \sqrt{\frac{1}{4} + n^2 - n - 2m} \rfloor$, and an integer $k \geq 0$, returns an induced subgraph G_0 of G such that $\alpha(G) \leq p - k$ if and only if $\alpha(G_0) \leq p - k$.

Theorem There exists an $O(3^{3k+1}kn)$ fpt-algorithm (fixed parameter tractable) to decide whether $\alpha(G_0) \leq p - k$.

Corollary Given as an input a graph G with n vertices, m edges, $p := \lfloor \frac{1}{2} + \sqrt{\frac{1}{4} + n^2 - n - 2m} \rfloor$, and an integer $k \geq 0$, it can be decided in time $O(3^{3k+1}kn)$ whether $\alpha(G) \leq p - k$.

References:

[1] P. Hansen and M. L. Zheng, Sharp bounds on the order, size, and stability number of graphs, *Networks* **23(2)** (1993) 99–102.

Fixed Parameter Tractable Algebraic Algorithm for a Hamiltonian Graph

Monday
15.20-15.40

Irene Sciriha

University of Malta, Malta

irene.sciriha-aquilina@um.edu.mt

To determine if a connected graph is Hamiltonian is NP–complete. We present a fixed parameter tractable algorithm that, given the cyclomatic number of the graph, we conclude whether the graph is Hamiltonian or not. The test depends on the presence of a discriminating vector in the nullspace of the adjacency matrix of the subdivision of the graph, derived from its walk matrix. The complexity of the algorithm on an n-vertex graph with m edges is $O(m - n)^4$.

On types of topological indices with respect to their edge contribution function

Tuesday
12.00-12.20

Jelena Sedlar

University of Split, Croatia

jsedlar@gradst.hr

(*joint work with* Damir Vukičević)

We introduce the ordering of tree graphs so that the star S_n is minimal, while the path P_n is maximal. Topological indices are defined to be of Wiener or anti-Wiener type, if they are increasing or decreasing functions with respect to the introduced ordering. Obviously, this leads to the simple corollary that the minimal graph for indices of Wiener type is S_n and the maximal graph is P_n. For indices of anti-Wiener type the reverse holds. Then we introduce a simple criterion on an edge contribution function of a topological index which enables us to establish if a topological index is of Wiener or anti-Wiener type. Finally, we apply our result to several generalizations of the Wiener index, such as modified Wiener indices, variable Wiener indices and Steiner $k-$Wiener index.

Nullity of Zero Divisor Graph of a Polynomial Ring

Thursday
11.20-11.40

Khidir R. Sharaf

University of Zakho, Iraq

khidirsharaf@yahoo.com

(*joint work with* Nihad T. Sarhan)

The zero divisor graph $\Gamma(R_n(Z_k[x]))$ of a polynomial quotient ring $R_n(Z_k[x]) = Z_k[x]/(M)$, where M is the principal ideal generated by the polynomial $f(x) = x^{n+1}$ and Z_k the ring of integers modulo k, is introduced.

Euler's phi function $\phi(x)$ is used to determine the above mentioned zero divisor graph. Also an infinite class of pairs of non-isomorphic rings with the same order having isomorphic zero divisor graphs are created.

Loop zero divisor graphs $\Gamma^o(R)$ for a commutative ring R are found, and some known results for graphs are generalized to loop zero divisor graphs such as: cluster lemma, end vertex corollary, and high zero sum weighting. These new generalized results are applied to evaluate the nullity of the looped zero divisor graph and some of its invariants.

Finally, if $\Gamma(R)$ is a complete graph, then $(Z(R))^2 = 0$ or $R = Z_2 \times Z_2$. While, if $R \neq Z_2 \times Z_2$ with $2 \notin Z^*(R)$, then $\Gamma^o(R)$ is a totally looped complete graph if and only if $(Z(R))^2 = 0$.

A unifying approach to constructions of Cayley graphs asymptotically approaching the Moore bound for diameters 2 and 3

Thursday
15.40-16.00

Jana Šiagiová

Slovak University of Technology, Slovakia

siagiova@math.sk

(*joint work with* Martin Bachratý and Jozef Širáň)

The degree-diameter problem is to determine the largest order of a graph of a given maximum degree d and a given diameter k. The order of such a graph cannot exceed the Moore bound $M(d, k)$ that has the form $d^k + O(d^{k-1})$ for fixed k and $d \to \infty$. It is well known (cf. [4]) that for $d \geq 3$ and $k \geq 2$ there are graphs of order $M(d, k)$ only if $k = 2$ and d is $3, 7$ and possibly 57. On the other hand, quotients of incidence graphs of generalized triangles, quadrangles, and hexagons by polarity [3] yield graphs of maximum degree d and order $d^2 - O(d)$ for diameter 2 if $d-1$ is a prime power, order $d^3 - O(d^2)$ for diameter 3 whenever $d-1$ is a power of 2, and order $d^5 - O(d^4)$ for diameter 5 if $d-1$ is a power of 3. In all these cases, however, the graphs are not even regular, and for diameter 2 they have been proved to be far from vertex-transitive even after regularization [1].

This raised the question of whether or not there are vertex-transitive or Cayley graphs of diameter $k \in \{2,3,5\}$ for infinite sets of degrees d whose order divided by $M(d,k)$ tends to 1 as $d \to \infty$. Constructions of such families of Cayley graphs were discovered recently in [5] for diameter 2 (with a geometric approach leading to the same family developed in [1]), and in [2] for diameter 3; the case of diameter 5 is still open.

In this contribution we outline a unifying approach to both constructions, based on identification of suitable orbits of groups acting on graphs.

References:

[1] M. Bachratý and J. Širáň, Polarity graphs revisited, *Ars Math. Contemp.* **8** (2015) 55–67.

[2] M. Bachratý, J. Šiagiová and J. Širáň, Asymptotically approaching the Moore bound for diameter three by Cayley graphs, *submitted* (2015).

[3] Ch. Delorme, Examples of products giving large graphs with given degree and diameter, *Discrete Appl. Math.* **37-38** (1992) 157–167.

[4] M. Miller and J. Širáň, Moore graphs and beyond: A survey of the degree/diameter problem, *Electr. J. Combin.* **20** (2013), Dynamic Survey DS14v2, 92 pp.

[5] J. Šiagiová and J. Širáň, Approaching the Moore bound for diameter two by Cayley graphs, *J. Combin. Theory Ser. B* **102** (2012) 470–473.

Distance Magic Labelings of Distance Regular Graphs

Wednesday
11.20-11.40

Rinovia Simanjuntak

Institut Teknologi Bandung, Indonesia

rino@math.itb.ac.id

(*joint work with* I. Wayan Palton Anuwiksa)

For an arbitrary set of distances $D \subseteq \{0, 1, \ldots, diam(G)\}$, a graph G is said to be D-magic if there exists a bijection $f : V(G) \to \{1, 2, \ldots, |V(G)|\}$ and a constant k such that for any vertex x, $\sum_{y \in N_D(x)}(y) = k$, where $N_D(x) = \{y \in V(G) | d(x,y) \in D\}$. We define a D-distance graph of a graph G, denoted by $\Delta_D(G)$, as the graph with vertex set $V(G)$ and edge set $\{\{x,y\} | d_G(x,y) \in D\}$.

We shall search for D-magic distance regular graphs for various D by using several methods, which include the spectrum of G and $\Delta_D(G)$.

Monday
15.20-15.40

Orientations in several rounds making k-cycles cyclic at least once

Gábor Simonyi

Rényi Institute of Mathematics, Hungarian Academy of Sciences, Hungary

simonyi.gabor@renyi.mta.hu

(*joint work with* Zita Helle)

Vera Sós asked in 1991 how many 3-edge-colorings of the complete graph K_n on n vertices are needed if every triangle is made 3-colored in at least one of them. Denoting this number with $h(n)$ it is proven in [2], that

$$\lceil \log_3(n-1) \rceil \leq h(n) \leq \lceil \log_2 n \rceil - 1$$

holds. The exact value is still unknown.

Here we consider an analogous question for orientations that we can fully answer. Let $t(n)$ denote the minimum number of orientations of the edges of K_n needed to make every triangle cyclically oriented in at least one of them. A simple local argument gives that $t(n) \geq \lceil \log_2(n-1) \rceil$. Unlike in the above case (where the lower bound is obtained by a similar argument), here this simple lower bound turns out to be sharp, that is, the following holds.

Theorem [1].

$$t(n) = \lceil \log_2(n-1) \rceil.$$

In fact, we prove the following more general statement.

Let $t(n,k)$ denote the minimum number of orientations of the edges of K_n on vertex set $\{v_1, \ldots, v_n\}$ such that for any k-subset $\{v_{i_1}, \ldots, v_{i_k}\}$ of the vertices with $i_1 < i_2 < \cdots < i_k$, the cycle $v_{i_1} - v_{i_2} - \cdots - v_{i_k} - v_{i_1}$ is cyclically oriented in at least one of them. Then the following generalization of the above theorem is true.

Theorem [1]. For every $n \geq k \geq 3$ we have

$$t(n,k) = \left\lceil \log_2 \frac{n-1}{k-2} \right\rceil.$$

References:

[1] Z. Helle, G. Simonyi, Orientations making k-cycles cyclic, *Graphs and Combinatorics* **32** (2016) 2415–2423.

[2] J. Körner, G. Simonyi, Trifference, *Studia Sci. Math. Hungar.* **30** (1995) 95–103; also in: W. A. Deuber and V. T. Sós (eds.), *Combinatorics and its Applications to the Regularity and Irregularity of Structures*, Akadémiai Kiadó, Budapest (1995).

Extremal questions for permutations

Monday
16.40-17.00

Dániel Soltész

MTA Rényi Institute, Hungary

soltesz.daniel@renyi.mta.hu

(*joint work with* István Kovács)

This talk will be about questions of the following type. Suppose that there is a given compatibility relation between two permutations.

What is the maximal number of pairwise compatible permutations?

For different compatibility relations we can get quite different questions. We will mention a few examples like intersecting, t-intersecting, reversing and colliding permutations. Even the maximal number of Hamiltonian paths in tournaments can be regarded as a question of this type. By the natural correspondence between permutations and Hamiltonian paths some of these questions can be formulated using purely graph theoretic terms. In this talk we are interested in ones that can be formulated using undirected Hamiltonian paths as follows.

What is the maximal number of pairwise compatible Hamiltonian paths of K_n?

Körner, Messuti and Simonyi observed that it is easy to determine the maximal number of Hamiltonian paths of K_n where each pairwise union must contain an odd cycle. They asked whether this is the same as the maximal number of Hamiltonian paths of K_n where each pairwise union must contain a triangle. Their question was motivated by examples for small n and several other questions in combinatorics where asking something for odd cycles and triangles produces the same outcome. We managed to answer the question affirmatively. In this talk we present the upper bound (that is quite simple) and the main ideas of the construction. Our method can be used to obtain similar but only asymptotic results for other odd cycles instead of triangles. We conclude the talk with several open questions. Most of the results mentioned can be found in [1].

References:

[1] I. Kovács, D. Soltész, Triangle-different Hamiltonian paths, *arXiv:1608.05237*

Thursday
12.00-12.20

On the radius and the attachment number of tetravalent half-arc-transitive graphs

Primož Šparl

University of Ljubljana and University of Primorska, Slovenia

primoz.sparl@pef.uni-lj.si

It is well known that the action of a half-arc-transitive (sub)group of automorphisms (that is vertex- and edge- but not arc-transitive) on a given graph Γ induces two paired natural orientations of the edge-set of Γ. In the case that Γ is tetravalent this gives rise to *alternating cycles* all of which have the same even length $2r$. Moreover, any two nonadjacent alternating cycles meet in the same number a of vertices. The parameters r and a, known as the *radius* and the *attachment number*, respectively, were introduced by Marušič [1] and play an important role in the investigation of tetravalent graphs admitting a half-arc-transitive (sub)group of automorphisms. For instance, the situation when $a = r$ is very well understood and all tetravalent half-arc-transitive graphs (the ones where the whole automorphism group of the graph is half-arc-transitive) with this property have been classified [1,3].

In this talk we present some new results regarding the relationship between the parameters r and a. For instance, it is well known that a always divides $2r$. However, the census of all tetravalent half-arc-transitive graphs up to order 1000, recently constructed by Potočnik, Spiga and Verret [2], shows that for all such graphs up to order 1000, the attachment number a in fact divides r (while in the case of arc-transitive graphs admitting a half-arc-transitive subgroup of automorphisms this is not true in general). We show that this phenomenon occurs at least in all tetravalent half-arc-transitive graphs (regardless of the order) for which a is twice an odd number. In addition, we completely characterize the (arc-transitive) tetravalent graphs admitting a half-arc-transitive subgroup of automorphisms with $r = 3$ and $a = 2$, and show that they in fact arise as certain non-sectional split covers, providing an interesting way of constructing these otherwise rather elusive covers.

References:

[1] D. Marušič, Half-transitive group actions on finite graphs of valency 4, *J. Combin. Theory Ser. B* **73** (1998) 41–76.

[2] P. Potočnik, P. Spiga, G. Verret, A census of 4-valent half-arc-transitive graphs and arc-transitive digraphs of valence two, *Ars Math. Contemp.* **8** (2015) 133–148.

[3] P. Šparl, A classification of tightly attached half-arc-transitive graphs of valency 4, *J. Comb. Theory, Ser. B* **98** (2008) 1076–1108.

On the zero forcing number of graphs with given girth and minimum degree

Monday
17.40-18.00

Sudeep Stephen

The University of Newcastle, Australia

sudeep.stephen@uon.edu.au

(*joint work with* Thomas Kalinowski and Randy Davila)

For a two-coloring of the vertex set of a simple graph $G = (V, E)$, consider the following color-change rule: a red vertex is converted to blue if it is the only red neighbour of some blue vertex. A vertex set $S \subseteq V$ is called *zero-forcing* if, starting with the vertices in S blue and the vertices in the complement $V \setminus S$ red, all the vertices can be converted to blue by repeatedly applying the color-change rule. The minimum cardinality of a zero-forcing set for the graph G is called the *zero-forcing number* of G, denoted by $Z(G)$. The main contribution of this paper is to prove the following conjecture originally posed by Davila and Kenter in [2], and partially resolved in [1, 2, 3, 4]; namely, if G is a graph with minimum degree $\delta \geq 2$ and girth $g \geq 3$, then $Z(G) \geq \delta + (\delta - 2)(g - 3)$.

References:

[1] R. Davila and M. Henning, The forcing number of graphs with a given girth, *arXiv:1610.08435* (2016).

[2] R. Davila and F. Kenter, Bounds for the zero forcing number of a graph with large girth, *Theory and Applications of Graphs* **2** (2015), 1–10.

[3] M. Gentner, L. D. Penso, D. Rautenbach, and U. S. Souzab, Extremal values and bounds for the zero forcing number, *Discrete Applied Mathematics* **214** (2016), 196–200.

[4] M. Gentner and D. Rautenbach, Some bounds on the zero forcing number of a graph, *arXiv:1608.00747* (2016).

Thursday
17.20-17.40

Mixed metric dimension of graphs

Andrej Taranenko

University of Maribor, Slovenia

andrej.taranenko@um.si

(*joint work with* Aleksander Kelenc, Dorota Kuziak and Ismael G. Yero)

Let $G = (V, E)$ be a connected graph. A vertex $w \in V$ distinguishes two elements (vertices or edges) $x, y \in E \cup V$ if $d_G(w, x) \neq d_G(w, y)$. A set S of vertices in a connected graph G is a mixed metric generator for G if every two elements (vertices or edges) of G are distinguished by some vertex of S. The smallest cardinality of a mixed metric generator for G is called the mixed metric dimension and is denoted by $\mathrm{mdim}(G)$. In this talk we consider the structure of mixed metric generators and characterize graphs for which the mixed metric dimension equals the trivial lower and upper bounds. We also present results about the mixed metric dimension of some families of graphs and provide an upper bound with respect to the girth of a graph. Finally, we present some results about the complexity of the problem of determining the mixed metric dimension of a graph, which is NP-hard in the general case.

Tuesday
11.40-12.00

Edge-colourings of graphs - a personal view

Bjarne Toft

University of Southern Denmark, Denmark

btoft@imada.sdu.dk

In 1977 Stanley Fiorini and Robin Wilson published a book *Edge-colourings of graphs* (Pitman Research Notes in Mathematics 16) containing the first comprehensive exposition of edge-colouring theory. The book is stimulating and useful, containing a wealth of information, including a complete bibliography of more than 200 works related to edge-colouring up to 1977. In 2012 a second book devoted to edge-colouring appeared (Stiebitz et al., *Graph Edge Coloring*, Wiley 2012), and there is also a more recent interesting survey by Jessica McDonald in (*Topics in Chromatic Graph Theory*, ed. L.W. Beineke and R.J. Wilson, Cambridge Univ. Press 2015).

Starting from the Fiorini-Wilson book the further development, leading to the Stiebitz et al. book, will be highlighted, in particular the (somewhat neglected) work of Ram Prakash Gupta and the very important results and conjectures of Paul Seymour. The conjecture of Mark Goldberg (also called the Goldberg-Seymour Conjecture) (1970s) is still the most important unsolved problem in the area. Other famous problems are the Berge-Fulkerson Conjecture (late 1970s) and the List-Chromatic-Index Conjecture (1980s).

Expanding expressive power of MSO logic: algorithms for dense graph classes

Thursday
16.20-16.40

Tomáš Toufar

Charles University, Czech Republic

toufi@iuuk.mff.cuni.cz

(*joint work with* Dušan Knop, Martin Koutecký and Tomáš Masařík)

A celebrated theorem of Courcelle [1] states that every property definable in monadic second order logic (MSO) sentence can be decided in linear time on classes of graphs of bounded treewidth. Inspired by this result, efficient model checking algorithms for different classes of graphs (e.g. of bounded neighborhood diversity) have emerged [3].

An interesting question is whether we can expand the expressive power of MSO logic while still retaining efficient algorithms. Different extensions have been studied before. One such example is CardMSO logic introduced by Ganian [2]. In this extension, we are allowed to place linear restrictions on the cardinalities of certain sets. Another important example are *local cardinality constraints* introduced by Szeider [4]. Here we are allowed to restrict the size of $X \cap N(v)$ at each vertex.

We provide a unified approach to those extensions and we allow to use both kinds of constraints at once. Furthermore, we study complexity of the kind of constraints used by Ganian when we drop the linearity requirement.

We give an FPT algorithm with respect to neighborhood diversity for the case where both global and local constraints are linear. To complement this, we show that model checking for nonlinear CardMSO constraints is W[1]-hard with respect to neighborhood diversity. Finally, we give an XP algorithm (running time $n^{O(\mathrm{nd}(G))}$) for model checking for formula with both global and local constraints in their general form.

References:

[1] B. Courcelle, The monadic second-order logic of graphs. I. Recognizable sets of finite graphs, *Information and Computation*, **85 (1)** 12–75.

[2] R. Ganian, J. Obdržálek, Expanding the Expressive Power of Monadic Second-Order Logic on Restricted Graph Classes, *In Proc. of IWOCA 2013*, LNCS **8288** (2013) 164–177.

[3] M. Lampis, Algorithmic Meta-theorems for Restrictions of Treewidth, *Algorithmica* **64 (1)** 19–37

[4] S. Szeider, Monadic Second Order Logic on Graphs with Local Cardinality Constraints, *In Proc. of MFCS 2008*, LNCS **5162** (2008) 601–612.

Thursday
16.40-17.00

Structural Properties of Resonance Graphs

Niko Tratnik

University of Maribor, Slovenia

niko.tratnik@um.si, niko.tratnik@gmail.com

(*joint work with* Martina Berlič, Tomislav Došlić, Dong Ye and Petra Žigert Pleteršek)

Let G be a molecular graph. The resonance graph $R(G)$ is the graph whose vertices are the perfect matchings of G, and two perfect matchings are adjacent whenever their symmetric difference forms a hexagon of G. The concept of the resonance graph appears quite naturally in the study of perfect matchings of molecular graphs of hydrocabons that represent Kekulé structures of corresponding hydrocarbon molecules. The resonance graph of a molecular graph carries many important information on Kekulé structures.

The concept of the resonance graph has been introduced for different families of molecular graphs, for example benzenoid systems, carbon nanotubes, and fullerenes.

In the talk, some properties of resonance graphs will be considered, such as bipartiteness, connectedness, distributive lattice structure, equality of the Zhang-Zhang polynomial and the cube polynomial, etc. Moreover, the differences according to the specific family of molecular graphs will be presented.

Thursday
16.40-17.00

Searching for digraphs with small excess

James Tuite

Open University, UK

james.tuite@open.ac.uk

(*joint work with* Grahame Erskine)

The directed degree/diameter problem concerns the optimisation of the order of a digraph G with maximum out-degree d and diameter k. This problem has applications in the design of efficient interconnection networks. A natural upper bound on such a digraph is the Moore bound $M(d,k) = 1 + d + d^2 + ... + d^k$. A digraph will meet this upper bound if and only if it is out-regular with degree d and for any pair of vertices u, v there is exactly one directed path of length $\leq k$ from u to v. Unfortunately, this is possible only in trivial cases [1]. It is therefore of interest to investigate digraphs that in some sense approximate Moore digraphs.

Many authors have studied large digraphs with degree d and diameter k in which paths of length $\leq k$ between two vertices are not necessarily unique. An alternative approach which is receiving increasing attention is to ask for digraphs with minimum degree d which are k-geodetic (i.e. between any pair of vertices there is at most one $\leq k$-path) and have order $M(d,k)+\epsilon$ for some small *excess* $\epsilon > 0$ [2]; this can be viewed as an analogue of the undirected degree/girth problem. In this talk I will review existing results and present new lower bounds on such digraphs and some methods of constructing small k-geodetic digraphs.

References:

[1] W. G. Bridges, S. Toueg, On the impossibility of directed Moore graphs, *J. Combinatorial Theory B* **29** (1980) 339–341.

[2] A. Sillasen, On k-geodetic digraphs with excess one, *Electron. J. Graph Theory and Applications 2* **2** (2014) 150–159.

Minimal alliances in graphs

Thursday
11.20-11.40

Zsolt Tuza

MTA Rényi Institute and University of Pannonia, Hungary

tuza@dcs.uni-pannon.hu

(*joint work with* Cristina Bazgan and Henning Fernau)

As introduced in [1], a nonempty set S of vertices in a graph $G=(V,E)$ is a defensive alliance if every vertex $v \in S$ satisfies the condition

$$|N[v] \cap S| \geq |N[v] \cap (V \setminus S)|$$

where $N[v]$ denotes the closed neighborhood of v. The notion of strong defensive alliance is obtained by replacing $N[v]$ with $N(v)$, the open neighborhood of v. We study the maximum of $|S|$ for alliances S which are locally minimal ($S \setminus \{v\}$ is not an alliance for any $v \in S$), or globally minimal (no proper subset of S is an alliance).

Reference:

[1] P. Kristiansen, S. M. Hedetniemi and S. T. Hedetniemi, Alliances in graphs, *Journal of Combinatorial Mathematics and Combinatorial Computing* **48** (2004) 157–177.

On the minimum degree of minimally 1-tough graphs

Monday
11.40-12.00

Kitti Varga

Budapest University of Technology and Economics, Hungary

vkitti@cs.bme.hu

(*joint work with* Gyula Y. Katona and Dániel Soltész)

Let $\omega(G)$ denote the number of components of a graph G. A graph G is called t-tough for a positive real number t, if $\omega(G-S) \leq |S|/t$ for any cutset S of G. A graph G is said to be minimally t-tough, if it is t-tough, but removing any of its edges the resulting graph is no longer t-tough. Mader proved that every minimally k-connected graph has a vertex of degree k. Kriesell conjectured an analogue of Mader's theorem: every minimally 1-tough graph has a vertex of degree two. First, we show that the family of minimally t-tough graphs is rich, i.e. any graph can be embedded as an induced subgraph into a minimally t-tough graph. Our main result is that every minimally 1-tough graph of order n has a vertex of degree at most $n/3+1$. We also examine the conjecture in a special case. Matthews and Sumner proved that a noncomplete claw-free graph is $2t$-connected if and only if it is t-tough. Using this theorem we show that minimally 1-tough claw-free graphs are cycles, which implies that in this graph family the conjecture is trivially true.

Wednesday
11.20-11.40

Self-Similar Polygonal Tiling

Andrew Vince

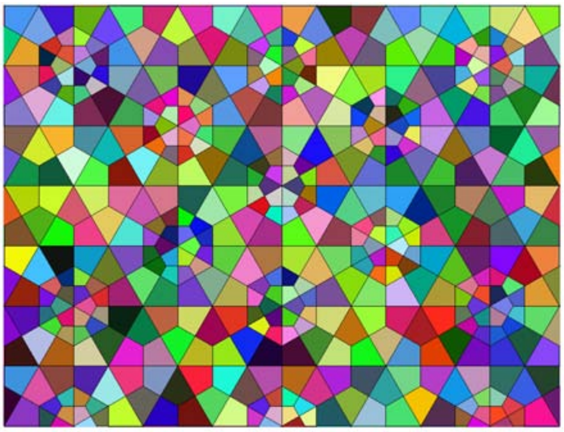
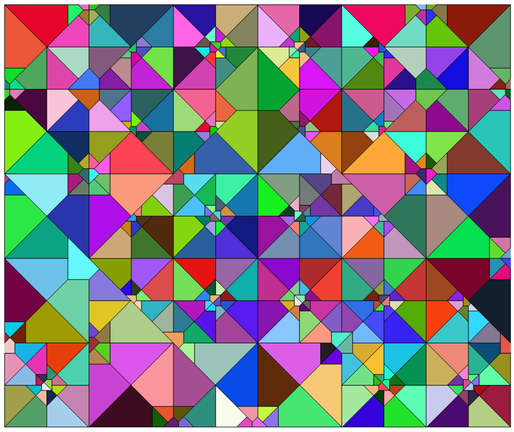
University of Florida, USA

avince@ufl.edu

(*joint work with* Michael Barnsley)

The goal of the talk is to explain the combinatorics underlying tilings like those in the figures below. In each tiling, the individual polygonal tiles are pairwise similar, and there are only finitely many up to congruence. Each tiling is *self-similar*. None of the tilings are periodic, yet each is *quasiperiodic*. Our method, based on rooted labeled trees, is used to construct such tilings. It is a scheme that extends and simplifies previous tiling constructions.

In the left figure, there are two similar tile shapes; in the right figure there are 6. In the tiling of the entire plane, the part shown in the figure appears "everywhere," the phenomenon known as quasiperiodicity or repetitivity. Quasiperiodicity, less stringent than periodicity, has gained considerable attention since the 1984 Nobel Prize winning discovery of quasicrystals by Shechtman, Blech, Gratias, and Cahn. Define a patch of a tiling T as a subset of T whose union is a topological disk. A

tiling of the plane is *quasiperiodic* if, for any patch U, there is a number $R > 0$ such that any disk of radius R contains, up to congruence, a copy of U. The tiling T in the figure is *self-similar* in that there exists a similarity transformation ϕ of the plane such that, for each tile $t \in T$, the "blown up" tile $\phi(t) = \{\phi(x) : x \in t\}$ is the disjoint union of the original tiles in T. In each of the two examples, there are uncountably many such tilings using the same set of tiles.

There is a cornucopia of tilings of the plane possessing some sort of regularity. The mathematical literature is replete with papers on the subject, for example the tilings by regular polygons dating back at least to J. Kepler, tilings with large symmetry group as studied by Grünbaum and Shephard and many others, and the aperiodic Penrose tilings and their relatives. Self-similarity, in one form or another, has been intensely studied over the past few decades — arising in the areas of fractal geometry, Markov partitions, symbolic dynamics, radix representation, and wavelets. In this talk, a novel method for the construction of self-similar polygonal tilings based on labeled rooted trees will be discussed.

Non-adaptive versions of combinatorial group testing and majority problems

Friday
11.40-12.00

Mate Vizer

Alfréd Rényi Mathematical Institute, Hungary

vizermate@gmail.com

(*joint work with* Dániel Gerbner)

In the most basic *model* of *combinatorial group testing* Questioner needs to find a special element (called *defective*) $x \in [n]$ by asking a minimal number of *queries* of type "is $x \in F \subset [n]$?". The model is *non-adaptive*, if Questioner has to pose all the queries at the beginning. The authors of [4] posed a new non-adaptive model, namely they assume that any element knows the answer for those queries that contain it, and require different goals to achieve by the elements. We investigate this kind of models and prove asymptotically sharp results on the cardinality of the optimal query sets using results from [2] and [5].

A related search problem is the so called *majority problem*, where we are given $[n]$ (we call them indexed *balls*), each $i \in [n]$ colored in some way unknown to us, and we would like to find a majority ball or to show that there is no majority ball by asking *queries* about subsets of $[n]$ ($i \in [n]$ is called *majority ball* if there are more than $\frac{n}{2}$ balls in the input set that have the same color as i). Again, a model is *non-adaptive*, if we have to pose all the queries at the beginning. For an intodutory survey on majority problems see [1] and for more recent results see [3]. We provide upper and lower bounds on the cardinality of optimal query sets for non-adaptive models, where the cardinality of the queries is greater than 2.

References:

[1] M. Aigner, Variants of the majority problem, *Discrete Applied Mathematics*, **137**(1) (2004) 3–25.

[2] P. Frankl and Z. Füredi, Union-free hypergraphs and probability theory, *European Journal of Combinatorics* **5.2** (1984) 127–131.

[3] D. Gerbner, B. Keszegh, D. Pálvölgyi, B. Patkós, M. Vizer, and G. Wiener, Finding a non-minority ball with majority answers, *Discrete Applied Mathematics*, **219** (2017) 18–31.

[4] J. Tapolcai, L. Rónyai, É. Hosszu, L. Gyimóthi, P.-H. Ho, S. Subramaniam, Signaling Free Localization of Node Failures in All-Optical Networks, *IEEE Transactions on Communications* **64**(6) (2016) 2527–2538.

[5] L. M. Tolhuizen, New rate pairs in the zero-error capacity region of the binary multiplying channel without feedback, *IEEE Transactions on Information Theory* **46**(3) (2000) 1043–1046.

Monday
12.20-12.40

Flag-transitive block designs with automorphism group S_n wr S_2

Tanja Vučičić

University of Split, Croatia

vucicic@pmfst.hr

(*joint work with* Snježana Braić and Joško Mandić)

Flag-transitive designs are designs that have an automorphism group acting transitively on the set of ordered pairs of incident points and blocks. They are interesting from the geometrical point of view for admitting a comparatively large automorphism group. In their construction different methods involving finite permutation groups are applicable.

Theoretically, each nontrivial flag-transitive block design can be observed as the substructure of a flag-transitive design whose full automorphism group is a maximal subgroup of a symmetric or an alternating group. Accordingly, starting from that design and its full automorphism group (of a rather large size), one can obtain flag-transitive designs by descending down the lattice of subgroups of the full group, considering ever smaller automorphism groups and then peeling back the arising corresponding substructures by an appropriately established procedure. An example of a group standing on the top of this procedure is the wreath product $S_n\ wr\ S_2$ in product action, which is a maximal subgroup for $n \geq 5$.

In this talk we describe a construction of flag-transitive block designs with product action of the automorphism group $S_n\ wr\ S_2$, $n \leq 36$. With n becoming too large to admit direct calculations with all combinatorial possibilities in software package MAGMA, we developed a specific approach to the construction by using flag-transitive or weakly flag-transitive incidence structures in obtaining base blocks.

References:

[1] P.J. Cameron and C.E. Praeger, Block-transitive t-designs I: point-imprimitive designs, *Discrete Mathematics* **118** (1993) 33-43.

[2] P-H. Zieschang, Point Regular Normal Subgroups of Flag Transitive Automorphism Groups of 2-Designs, *Advances in Mathematics* **121** (1996) 102-123.

[3] D. Marušić and T. Pisanski, Weakly Flag-transitive Configurations and Half-arc-transitive Graphs, *Europ. J. Combinatorics* **20** (1999) 559-570.

[4] J. de Saedeleer et al., Core-free, rank two coset geometries from edge-transitive bipartite graphs, *Mathematica Slovaca* **64** (2014) 991-1006.

On the minimum vertex cover of generalized Petersen graphs

Wednesday
11.40-12.00

David Wang

Beijing Institute of Technology, China

glw@bit.edu.cn

(*joint work with* Dannielle Jin)

Suppose that $k \geq 4$ and $n/\gcd(n,k)$ is odd. By using elementary number theory, we show that the minimum vertex cover number of the generalized Petersen graph $P(n,k)$ equals to $n+2$ if and only if $n \in \{9,\ 2k+2,\ 3k-1,\ 3k+1\}$.

Monday
11.20-11.40

Congruences for the Number of Transversals and Rainbow Matchings

Ian Wanless

Monash University, Australia

ian.wanless@monash.edu

(*joint work with* Darcy Best)

Consider a Latin square L, or equivalently a proper edge colouring C_L of $K_{n,n}$ using n-colours. Let E_i be the number of perfect matchings that include exactly i different colours in C_L. Of greatest interest is E_n, the number of transversals of L, and the number of rainbow matchings in C_L.

We are interested in congruences involving the E_i and related quantities. In 1990, Balasubramanian proved that $E_n \equiv 0 \mod 2$ when $n \equiv 0 \mod 2$. We have a number of new results along the same lines; for example,

- $E_n \equiv 0 \mod 4$ when $n \equiv 2 \mod 4$, and
- $E_{2k} \equiv E_{2k-1} \mod 2$ for $1 \leqslant k \leqslant n/2$ when $n \equiv 0 \mod 2$.

We also have a number of conjectures of similar flavour. In the course of our investigations we discovered that the number of perfect matchings in a k-regular bipartite graph on $2n$ vertices is divisible by 4 when n is odd and $k \equiv 0 \mod 4$ (was this previously known?).

Tuesday
11.20-11.40

Early milestones in the edge-colouring of graphs

Robin Wilson

Open University and the London School of Economics, UK

r.j.wilson@open.ac.uk

In this talk I present five 'milestone' papers in the edge-colouring of graphs that preceded the work of Stanley Fiorini and others in the 1970s. These papers were written by Peter Guthrie Tait (1880), Dénes König (1916), Claude Shannon (1949), and V. G. Vizing (1964, 1965).

Games of vertex coloring

Thursday
16.20-16.40

Wing Hong Tony Wong

Kutztown University of Pennsylvania, USA

wong@kutztown.edu

(*joint work with* Diego Manzano-Ruiz)

There are many variations of graph coloring games. In this project, we discuss a scenario where Alice and Barbara take turns to color the vertices of a given graph, with Alice starting first, so that no adjacent vertices share the same color. The first player who is unable to color a vertex loses the game. We consider the following two versions: 1. Alice uses color A and Barbara uses color B; 2. both of them use a common color C. Under both versions, we examine various families of graphs and determine which player has a winning strategy. Examples of such families include paths, cycles, rectangular grids, triangular grids, and Cayley graphs, etc. We also prove some general assertions about all graphs. These games are closely related to the Game of Col and Dawson's Chess Game studied by Berlekamp, Conway, and Guy.

References:

[1] E. Berlekamp, J. Conway, and R. Guy, *Winning Ways for your Mathematical Plays* (second edition), CRC Press, Roca Raton, FL (2001).

Colouring Σ-Hypergraphs

Tuesday
17.20-17.40

Christina Zarb

University of Malta, Malta

christina.zarb@um.edu.mt

(*joint work with* Yair Caro and Josef Lauri)

Let $V = \{v_1, v_2, ..., v_n\}$ be a finite set, and let $E = \{E_1, E_2, ..., E_m\}$ be a family of subsets of V. The pair $H = (V, E)$ is called a *hypergraph* with vertex-set $V(H) = V$, and with edge-set $E(H) = E$. When all the subsets are of the same size r, we say that H is an *r-uniform hypergraph*. Hypergraphs are generally viewed as a generalisation of graphs, which are essentially 2-uniform hypergraphs. Several important properties and results related to graphs have been generalised within the context of hypergraphs, with interesting and sometimes unexpected outcomes. In this talk we focus on hypergraph colouring. We look at different types of hypergraph colouring, in particular Voloshin colourings, NMNR-colourings and (α, β)-colourings. Perhaps one of the most interesting aspect of these types of colourings is that the chromatic spectrum can be broken, or have a gap, that is there can exist $k_1 < k_2 < k_3$ such that a hypergraph H is k_1- and k_3-colourable, but not k_2-colourable.

We introduce a family of r-uniform hypergraphs — the σ-hypergraphs, and we use these hypergraphs to investigate gaps in the chromatic spectrum, presenting interesting results about the existence or non-existence of gaps in the chromatic spectrum of σ-hypergraphs.

We then investigate the phenomenon of gaps further by introducing the notion of Q-colourings, which can be used to describe various different types of colourings of hypergraphs. We look at the necessary conditions required for the existence, or not, of gaps in the chromatic spectrum of a particular type of colouring. For this investigation, we extend the construction of σ-hypergraphs to the more general Σ-hypergraphs.

Friday
12.20-12.40

Counting Symmetric Bracelets

Yuliya Zelenyuk

University of the Witwatersrand, South Africa

yuliya.zelenyuk@wits.ac.za

An r-ary bracelet of length n is an equivalence class of r-colorings of vertices of a regular n-gon, taking all rotations and reflections as equivalent. A bracelet is symmetric if a corresponding coloring is invariant under some reflection. We show that the number of symmetric r-ary bracelets of length n is $\frac{1}{2}(r+1)r^{\frac{n}{2}}$ if n is even, and $r^{\frac{n+1}{2}}$ if n is odd [1,2].

References:

[1] Y. Gryshko, Symmetric colorings of regular polygons, *Ars. Combinatorica* **78** (2006) 277–281.

[2] Ye. Zelenyuk and Yu. Zelenyuk, Counting symmetric bracelets, *Bull. Aust. Math. Soc.* **89** (2014) 431–436.

The Sierpiński product of graphs

Thursday
16.20-16.40

Sara Sabrina Zemljič

University of Iceland, Iceland

sara.zemljic@gmail.com

(*joint work with* Jurij Kovič, Tomaž Pisanski, Arjana Žitnik)

The family of Sierpiński graphs has been studied very often in the past few decades for different reasons. One of them is definitely their relation to the famous Sierpiński triangle fractal and their fractal-like structure. The main building blocks of Sierpiński graphs are complete graphs and each next iteration is built in the fractal-like manner of a complete graph. This idea was recently generalized to generalized Sierpiński graphs, where instead of initially taking a complete graph, we start with an arbitrary graph G. Next iterations are then build in the same manner as graph G is constructed.

We generalized this idea even further by defining a Sierpiński product of two arbitrary graphs G and H, where we take $|G|$ copies of graph H and connect these according to edges in graph G. So intuitively we get a graph with local structure like H, but global structure like G. That is if we contract all copies of H, we get a copy of graph G. As most graph products this can be applied on any number of factors and if all (say n) factors are complete graph K_p, then the resulting graph is the Sierpiński graph S_p^n.

In the talk I will describe the Sierpiński product and related constructions, list some of their basic properties and examples.

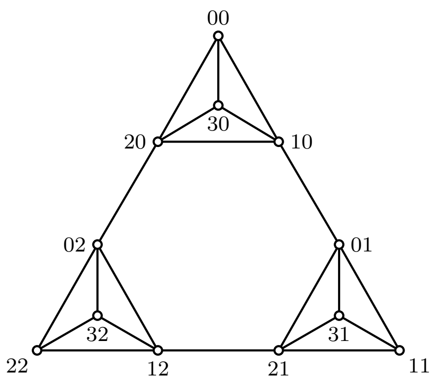


Figure 2: The Sierpiński product of C_3 and K_4.

Author Index

R

S