Viana, V., Nagy, D., Xavier, J., Neiva, A., Ginoulhiac, M., Mateus, L. & Varela, P. (Eds.). (2022). Symmetry: Art and Science | 12th SIS-Symmetry Congress [Special Issue]. *Symmetry: Art and Science*. International Society for the Interdisciplinary Study of Symmetry. 030-033. https://doi.org/10.24840/1447-607X/2022/12-02-030

MAURITS CORNELIS ESCHER: WHEN A SIMPLE EMPTY PLANE COMES TO LIFE

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Abstract: *In 1958, the Dutch graphic artist Maurits Cornelis Escher (1898-1972) published his book Regelmatige Vlakverdeling (The Regular Division of the Plane), including perfect tessellations where shapes or combinations of shapes interlock to cover the plane. In this paper, we analyse the geometry behind Escher's tessellations, transition devices, and an interesting interaction between Escher and the British mathematical physicist and philosopher Roger Penrose (1931-, Nobel Laureate in 2020).*

Keywords: Maurits Cornelis Escher, regular division of the plane, tessellations.

INTRODUCTION

The Dutch graphic artist Maurits Cornelis Escher (1898-1972) excelled in the representation of continuous transformations, use of ambiguities, manipulation of mirrors, optical illusions, regular division of the plane, infinity. Besides presenting an impeccable technique, the artist often used mathematics in his work, in particular some geometric concepts of considerable sophistication. For that reason, he is much appreciated by scientists and mathematicians. It is important to emphasize that his works are wonderful artistic objects, and, in the first analysis, it is because of that they should be appreciated. By producing very good art and excellent objects for geometers, M.C. Escher became a very popular artist.

Escher began his series of drawings *The Regular Division of the Plane* in 1936. These images are perfect tessellations where shapes or combinations of shapes interlock to cover the plane. One among some other sources of inspiration for this work was a visit to *Alhandra*, a 14th century Moorish castle near Granada, Spain. In 1958, Escher published his book *Regelmatige Vlakverdeling* (*The Regular Division of the Plane*), including several prints to demonstrate the concept. The series continued until the late 1960s, ending with the drawing 137.

In addition to the artistic and geometric challenge underlying the division of the plane, Escher obtained a tool to incorporate the idea of motion and transition in some of his works. Here, we analyse (1) the geometry behind Escher's tessellations, (2) transition devices, and (3) an interesting interaction between Escher and the British mathematical physicist and philosopher Roger Penrose (1931-, Nobel Laureate in 2020). The following sections succinctly develop the contents of each of these three parts.

LIFE BEGINS

An Escher's recurring theme (also a device) is the regular division of the plane (Schattschneider, 1990). With interlocking birds, fish, lizards, or other creatures, he made wallpapers (or fragments of wallpapers) and used these constructions to give the idea of motion, symmetry, or infinitude. About that, the artist wrote the following in 1958, cited in Schattschneider (1990, Preface):

> *At first, I had no idea at all of the possibility of systematically building my figures. I did not know any «ground rules» and tried, almost without knowing what I was doing, to fit together congruent shapes that I attempted to give form of animals. Gradually, designing new motifs became easier as a result of my study of the literature on the subject, as far this was possible for someone untrained in mathematics (…)*

In 1922, after leaving Haarlem, the artist embarked from Amsterdam to Spain. He travelled by train to some cities, and, in Granada, he visited Alhambra. He saw, for the first time, the decorative Moorish tilings and he was very much surprised. He wrote in his diary in 1922, as cited in Schattschneider, (1990, p. 9):

> *The strange thing about this Moorish decoration is the total absence of any human or animal form; even, almost any plant form. This is perhaps both a strength and a weakness at the same time.*

That is a very interesting comment, emphasizing both the strength of art and the power of abstract geometry. Later, in 1936, he visited Alhambra again with his wife. Both he and Jetta made nice sketches of Alhambra designs. After that, he started a series of drawings and, in 1958, he published his book *Regelmatige Vlakverdeling*, including several woodcuts prints to demonstrate his ideas. The series of drawings continued until the late 1960s, ending at drawing 137.

Mathematically speaking, the wallpaper group is a mathematical classification of a two-dimensional repetitive pattern, based on symmetries. A proof that there are exactly 17 distinct groups of possible patterns was first carried out by Evgraf Fedorov in 1891 (Conway, Burgiel & Goodman-Strauss, 2008). From Escher's point of view, Fedorov classification was not especially interesting, given that it considers the type of symmetry and not the conception of the tiles (an artist wants to make nice tiles!). Later, Heinrich Heesch and Otto Kienzle proposed a complete classification of 28 types of asymmetric tiles which tessellate (one tile in a monohedral tessellation of the plane – Heesch & Kienzle, 1963). Heesch types correspond to what Escher wanted, explaining the secrets of the tiles design. However, being much more recent, this classification was unknown to him. What is remarkable is the fact that Escher discovered, in his notebooks, 27 of the 28 possible Heesch types (missing only TCTGG). It is an outstanding example of interdisciplinarity, showing how art and mathematics can run together.

For example, *Vissen* (Fish, 1942, Drawing 55), is based on 60º-120º kites, shapes that tesselate. The fish is made by deforming two sides of a kite of that type; proceeding like that, in a sense, the fish is essentially a 60º-120º kite and, due to that, it tessellates. There is a pair of adjacent sides that relate through a 1/6 turn and there is a pair of adjacent sides that relate through a 1/3 turn. That explains the code C3C3C6C6 (Heesch code).

LIFE IS MOTION

In many of Escher's works, divisions of the plane change or evolve in some way. A well-known example is *Metamorphosis* III, a long print containing ingenious transitions between patterns, tilings, and realistic scenery. Escher would narrate it like a wonderful story. Escher's artistic work turns up 18 drawings employing some kind of *transition device*; in fact, it is possible to identify six categories of transition (Craig, 2008). In the simplest category, the tilings are of the same isohedral type and have congruent arrangements of vertices (points where three or more tiles meet).

THERE ARE EXCEPTIONS TO EVERYTHING IN LIFE

In 1962, Roger Penrose visited Escher in his house in Baarn. Throughout his career Penrose was fascinated by tessellations, a fascination that he shared with Escher, and, at some moment, he gave him a mysterious wooden puzzle. The shapes fitted together in many ways, but there was only one unique way in which they could be combined to create a tessellation containing all the pieces. It was a type of tessellation little studied by Escher, a kind of «exception». Escher accepted the challenge and solved the puzzle. However, solving the puzzle was not all. Penrose's basic shape inspired him to create his own artistic version of the puzzle! He produced the tessellation *Little Ghosts* in May 1971; it was the last drawing that Escher is known to have produced. A perfect interaction between a mathematician and an artist.

REFERENCES

Schattschneider, D. (1990). *Visions of Symmetry: Notebooks, Periodic Drawings and Related Work of M. C. Escher*. W.H. Freeman.

Conway, J.; Burgiel, H.. & Goodman-Strauss, C. (2008). *The Symmetries of Things*. AK Peters.

Heesch, H.; Kienzele, O. (1963). *Flächenschluss*. Springer.

Craig S. K. (2008). Metamorphosis in Escher's Art. *Bridges 2008 Proceedings*, 39-46.

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