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TREND, CYCLE, AND NON-LINEAR TRADE-OFF IN THE EURO AREA 1970-2001

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ABSTRACT

This paper presents new tests and estimates of the Phillips trade-off in the Euro Area, carried out in a unobserved components model with possibly non-linear Phillips and Okun relations, using quarterly aggregate data for the period 1970:I-2001:II. A concept of forward-looking near-rational expectations is introduced in the model, improving on the contradiction between rational expectations and evidence of inflation inertia. The Phillips curve turns out to be linear and its trade-off statistically significant, while non-linearity shows up in the Okun relation. The trend-cycle decompositions capture the main features of the Euro Area recent macroeconomic record.

KEYWORDS: Trend and cycle, Unobserved-components, Kalman filter, Phillips curve, Okun law, Near-rational expectations.

JEL CODES: E31, E32, C32.

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1. Introduction*

This paper addresses the trade-off between the unemployment gap and inflation changes, with an emphasis on the possibility of non-linearity. The trade-off is modelled in a simple unobserved components (UC) macro model based on the Phillips and Okun relations. Our tests for non-linearity, in both these macroeconomic relations, include the null hypothesis of linearity, and use model-consistent estimates of the unemployment and output gaps. Based on these tests results, we offer new estimates of the trend and cycle in the aggregate Euro Area during the period 1970-2001.

The use of the unobserved components model for macroeconomic trend-cycle decomposition tracks back to Watson (1986), Clark (1989) and Kuttner (1994). Our basic approach is, in turn, closer to Gordon's (1997) recent use of the time-varying framework to decompose the US unemployment rate into a stochastic trend (NAIRU) and cycle (gap) within a system including the Phillips relation, using the Kalman filter. A slowly time-varying NAIRU is consistent with the natural rate hypothesis as outlined in Friedman (1968) and seems to be necessary for a successful modelling of recent macro trends and cycles, in the US and other industrialised countries.

Following Gordon's seminal paper, several recent studies have estimated time-varying NAIRUs, and the associated gaps, from UC systems based on Phillips equations. For example: Laubach (2001) with G7 data; McAdam and McMorrow (1999) with US, Japan and Eur-15 data; Gerlach and Smets (1999) with data of EMU countries; Richardson *et al.* (2000) with data of OECD countries; Orlandi and Pichelman (2000) with annual Euro Area data; and Irac (2000), Estrada *et al.* (2000), and Meyler (1999), for some individual EMU countries.

At a more institutional level, OECD research has recently assumed that Kalman filter estimates of time-varying NAIRUs, modelled in Gordon-style frameworks, would be considered the benchmark for their trend-cycle decompositions of unemployment - see Boone (2000), Richardson *et al.* (2000), and OECD (2000, chapter V). Likewise, the Gordon framework has been used by the ECB research in the computation of the quarterly Euro Area Trend Unemployment Rate included in the Area Wide Model Database (AWMD) published in Fagan *et al.* (2001) - see also Fabiani and Mestre (2000).

Treating the NAIRU as a time-varying parameter seems particularly suited for the Euro Area case, in view of the persistent rise in European unemployment during the 1970s, 1980s and part of the 1990s. It does not seem realistic to try to explain and model such a phenomenon without admitting an increase in the equilibrium unemployment rate itself, as has been argued in several instances by Olivier Blanchard. Furthermore, there are reasons to believe that this process could now be under reversion, as the European NAIRU may presently be decreasing – see, for example, Blanchard and Wolfers (2000), and Blanchard and Giavazzi (2001).

In this context of time-varying NAIRU and stochastic trend real output, testing for non-linearity in the Euro Area Phillips and Okun relations is important on technical, theoretical and economic policy grounds. Technically, non-linearity tests provide useful information for the adequate specification of the measurement system in the unobserved components model. Theoretically, the evidence from non-linearity tests has crucial implications for our knowledge about the Euro Area monetary transmission mechanism. Politically, it is known that monetary policy operates and impacts on the economy quite differently under non-linearity, and, thus, it should be conceived and conducted differently if enough evidence of asymmetry is found.

Our empirical strategy is *ex-ante* unbiased and coherent, since we always nest the linear baseline model in the non-linearity tests and use model-consistent trends and gaps. Moreover, our framework improves the treatment of two contentious issues that have recently been highlighted within the Kalman filter estimation of stochastic NAIRUs - the unbiased estimation of the signal-to-noise ratio and the proper modelling of expectations in the Phillips equation.

The rest of the paper is planned as follows. In section 2 we discuss the relevance and implications of non-linearity in the inflation-unemployment trade-off. In section 3 we outline the unobserved components model and explain our empirical strategy, with special focus on the non-linearity tests. Section 4 summarises the empirical results for the Euro Area, 1970-2001, first discussing the findings from the non-linearity tests and then presenting the trend-cycle estimates (complemented with the appropriate confidence bands), obtained from the identified model. Section 5 offers some concluding remarks.

2. Addressing non-linearity

The hypothesis of a non-linear Phillips equation has solid theoretical roots, attractive empirical motivations, and important implications for monetary policy. These reasons are plenty to justify the recent up-surge of a renewed interest in the non-linearity hypothesis.

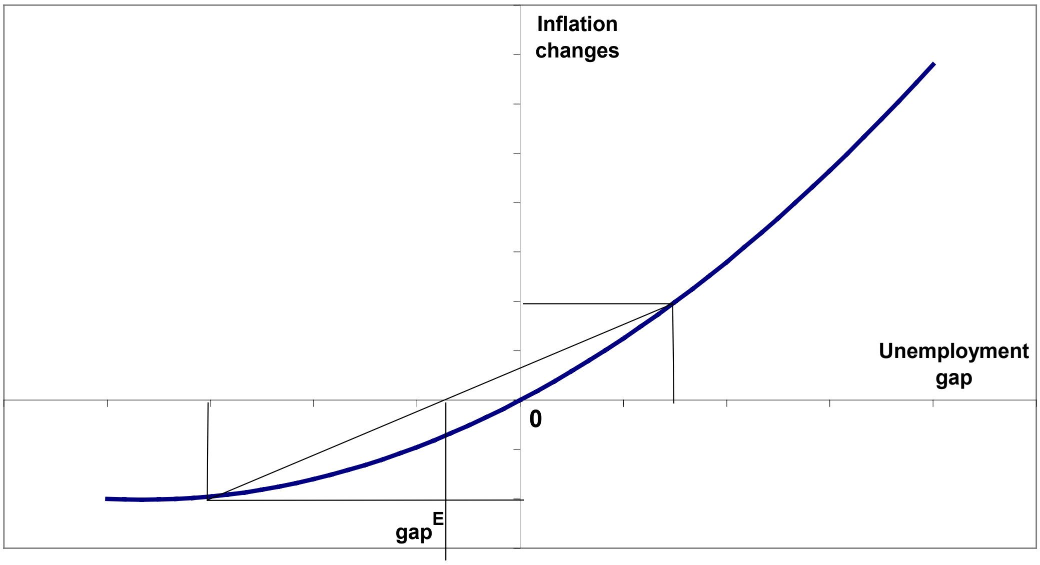
Theoretically, Ball, Mankiw and Romer (1988) and Ball and Mankiw (1994) included convex asymmetry of the Phillips relation as part of their New Keynesian approach. Most New Keynesian models of pricing behaviour imply a convex non-linearity in the short-run adjustment of prices to aggregate demand shocks. That is the case of the capacity constraint model, the menu costs and wage contracts theories, the downward nominal rigidity model and the efficiency wage model - see Dupasquier and Ricketts (1998) for a survey. The monopolistic competition model seems to be the only one to predict a concave non-linearity - see Eisner (1997) and Stiglitz (1997).

Empirically, non-linearity could explain many stylised facts of the post-War industrialised countries, like the ones observed for the US by De Long and Summers (1988), and the inflationary bias documented by Macklem (1995). A non-linear trade-off could also be behind the asymmetric effects of monetary policy in several countries, highlighted by Cover (1992), Karras (1996a, b), Karras and Stoke (1999a, b), Kaufmann (2001), and Peersman and Smets (2001). It could explain, as well, why some estimates of benefice ratios of inflation are significantly smaller than the sacrifice of disinflations, as noted by Filardo (1998). And it could be behind Weise's (1999) finding that demand shocks seem to have stronger output effects in recessions and stronger price effects in expansions.

In the specific case of the Euro Area there may be additional reasons that strengthen the case in favour of a possible non-linear Phillips relation. It has been argued that the aggregation of sectors of activity and regions with different cyclical positions, and different Phillips elasticity, may result in a non-linearity in the trade-off at the aggregate level - see Mayes and Viren (2000) and Demertzis and Hallett (1995, 1998).

The policy implications of the convex non-linearity derive from the fact that the inflationary effects of excess demand are, in that case, larger than the deflationary effects of an excess supply of the same magnitude. The chart below illustrates this idea. In the long-run, when there are no surprises or shocks, the natural rate vertical Phillips line would hold, and a null unemployment gap (defined in the chart as natural minus actual unemployment rate) would be consistent with no inflation changes. In the short-run convex Phillips curve, as inflation and economic activity vary stochastically, the inflation rate is symmetrically bounded around zero only if the unemployment gaps average to a negative value. This stochastic equilibrium locus is gap^E in the chart, which clearly does not coincide with the (0,0) locus of the deterministic equilibrium gap.[1]

ASYMMETRY IN THE CONVEX TIME-VARYING NAIRU PHILLIPS CURVE



Three main consequences arise for a monetary policy that assumes low and stable inflation as its main target. First, policy should aim at a negative gap, on average. Second, policy should be pre-emptive in acting against building inflationary pressures, as disinflation would be more costly than preventative anti-inflation. Third, policy should aim at reducing the variability of economic activity, in order to drive the average stochastic equilibrium gap closer to zero.

The empirical evidence on the non-linear trade-off has not settled the issue so far, in spite of recent refinements in the literature. In its earlier stages, tests allowed for a varying NAIRU (or trend output), but lacking a model-consistent estimation - like in Laxton *et al.* (1995), Turner (1995), Clark *et al.* (1995, 1996), Filardo (1998), Dupasquier and Ricketts (1997, 1998), Yates (1998), Pyythia (1999), and Mayes and Viren (2000).

Subsequently, the Phillips curve non-linearity has been considered in model-consistent frameworks - for instance, in Debelle and Laxton (1997), Clark and Laxton (1997), Debelle and Vickery (1998), Laxton *et al.* (1999), Faruqee *et al.* (1999), Isard *et al.* (2001), Meyler (1999), and Irac (2000). But these studies estimate non-linear models without formally testing that specification against the hypothesis of linearity. Furthermore, the non-linearity is specified as a function of the level of the unemployment rate, which, in the time-varying NAIRU framework, may not reflect the cyclical state of the economy.[2]

Our testing strategy, as detailed in the next section, overcomes both these problems in the literature. We let it to the data to identify the model as linear or non-linear, allowing for several alternative functional forms. And we model the possible asymmetry as a function of the unemployment gap.

We also consider and test for a non-linear Okun relation, jointly with the possibly non-linear Phillips curve. This is itself of interest - as stressed in recent literature by Mayes and Viren (2000), Lee (2000), Viren (2001), and Harris and Silverstone (2001) - and deepens the knowledge about the Euro Area cyclical behaviour. Moreover, it enables a more precise definition of the trade-off, by providing an indirect test of the hypothesis that its linearity/non-linearity depends upon the gap considered - unemployment or output. In our model, we have the opportunity to offer a first piece of evidence on this for the aggregate Euro Area, with the further advantage of using model-consistent output and unemployment gaps.

3. Unobserved components model

In this section we set up an unobserved components (UC) model by building up on Gordon's (1997), and explain how non-linearity can be considered in the measurement equations. But first, in order to clarify some aspects of the model, we address two preliminary issues, related to the Okun relationship and to the treatment of inflation expectations.

Preliminary issues: Okun equation, and expectations

When estimating his model, Gordon (1997) feared that the time-varying NAIRU would pick-up all the variation in the Phillips equation residual. It turned out, however, that many researchers faced precisely the opposite - the pile-up problem. In short, the maximum likelihood estimator of the variance of the innovation to a non-stationary unobserved component that has low true standard deviation is biased downwards because a large amount of probability in its distribution piles-up at zero. This compromises the estimation of the signal-to-noise ratio even when the model efficiently uses information of both unemployment and inflation rates - see, *inter alia*, Laubach (2001).

As suggested by Apel and Jansson (1999a, b), we address this difficulty by including an Okun equation in the system, relating the unemployment and output gaps, thus adding additional information to the estimation of the variance of the NAIRU innovation.[3] This extension of the basic Gordon measurement system has the additional advantage of allowing to test for non-linearity in the Okun equation, as well as in the Phillips', in the same model-consistent framework.

As for the treatment of inflation expectations in the Phillips equation, we face a research dilemma. On one hand we want to use a forward-looking and model-consistent expectations framework, like the Calvo-Rotemberg specification.[4] But, on the other hand, we do not overlook the evidence - as in Ball (1994 and 1997), Fuhrer and Moore (1995), and Estrella and Fuhrer (1998) - that rational expectations consistently fail in fitting the observed inflation inertia, to a point that led Mankiw (2000, page 23) to state that "the assumption of adaptive expectations is, in essence, what the data are crying out for."

We use Ball's (2000) concept of near-rational inflation expectations in order to bypass the dilemma. Ball's hypothesis is that real-world agents form inflation expectations considering only the past information on inflation, but use it optimally, identifying and estimating the best linear univariate forecasting model.

> "The deviation from rationality is the fact that forecasts are univariate: agents ignore relevant variables such as output and interest rates. Aside from this limitation, agents' forecasts are optimal: they use inflation data as best they can. Metaphorically, one can imagine firms who use Box-Jenkins techniques to select an ARIMA model for inflation, but who do not go to the added trouble of using multivariate techniques." (Ball, 2000, p. 9)

This can be interpreted as a limited-information rational expectations rule, in the spirit of Akerlof and Yellen (1985a, b). It describes agents that, faced with the high costs of gathering and processing the whole information set required for rational expectations, limit their information set and form forward-looking expectations solely on the basis of past inflation.

In applying this type of near-rational expectations, we compute, exogenously to the model, a series of expectations of inflation changes predicted with the best ARIMA model identified and estimated with Box-Jenkins methods for the deflator series, and then use it as data in the forward-looking Phillips equation.[5] This specification can be labelled as New Keynesian - since it is forward-looking -, but is also compatible with the inflation inertia observed in the current monetary regime.[6]

UC model

Our basic model is an extended version of the unobserved components framework recently popularised by Gordon (1997). The main extensions, just explained above, are the inclusion of the Okun equation (with the consequent transition equation for the output trend), and the treatment of forward-looking inflation expectations as near-rational.

It consists of the following three measurement (1 to 3) plus four state/transition (4 to 7) equations:[7]

$$\Delta\pi_t = E_t \Delta\pi_{t+1} + \gamma(u_t^n - u_t) + \omega S_t + \varepsilon_t^{phi} \tag{1}$$

$$y_t = y_t^p + \theta(u_t^n - u_t) + \varepsilon_t^{ok} \tag{2}$$

$$u_t = u_t^n - (u_t^n - u_t) \tag{3}$$

$$u_t^n = u_{t-1}^n + \mu_{t-1} + \varepsilon_t^n \tag{4}$$

$$\mu_t = \mu_{t-1} + \varepsilon_t^{\mu} \tag{5}$$

$$u_t^n - u_t = \rho_1(u_{t-1}^n - u_{t-1}) + \rho_2(u_{t-2}^n - u_{t-2}) + \varepsilon_t^{gap} \tag{6}$$

$$y_t^p = y_{t-1}^p + g + \varepsilon_t^p \tag{7}$$

Equation (1) is a New Keynesian Phillips equation of the Calvo-Rotemberg type, where changes in inflation are explained by expectations of inflation changes, by aggregate demand pressure, and by transitory supply shocks. Inflation expectations are taken as near-rational, in the sense put forward above. We denote by E_t, as usual, expectations formed in period t, but we consider that the information then available reports up to period t-1. The demand pressure is proxied by the unemployment gap (here, NAIRU minus actual unemployment rate). Closing, so to say, Gordon's triangular determination of inflation, the inclusion of the third vertex - temporary supply shocks (S_t) - avoids misinterpretations of the NAIRU, the correct interpretation being the unemployment rate that stabilizes inflation once all the effects of temporary shocks have faded away.[8]

The second measurement equation is the Okun equation, relating the output gap (y^p is the trend in output) to the current unemployment gap, which is a well-known empirical regularity with a long tradition in macroeconomics.

The third measurement equation (3), stems from the seminal unobserved components model in Watson (1986), and decomposes the unemployment rate into the trend component (NAIRU) and the cyclical unemployment gap. Technically, it assures that the NAIRU and the gap sum-up to the observed unemployment rate.

The state-system models the dynamics of trend-cycle unemployment and output - equations (4) to (7). In equation (4), the NAIRU is assumed to follow a random walk, which is standard in the literature modelling its time-path without explaining it (as in here). The random walk process driving the NAIRU, includes, in turn, a random walk drift, which is described in equation (5). This is an appropriate procedure for modelling time series that exhibit, for some time, a specific trend, but may subsequently invert the process and drift in the opposite way.[9] As briefly argued above in the introduction, this may be well suited for the Euro Area, where equilibrium unemployment has been drifting up for some decades, but currently seems to be decreasing.

Equation (6) describes the unemployment gap as a stationary auto-regressive process of second order. This is the assumption made originally in Watson (1986), and then in Kuttner (1994), for the US output gap, and has been adopted thereafter in several studies dealing with unemployment gaps, such as Apel and Jansson (1999a, b), Rasi and Vilkari (1998), and Laubach (2001). Specifying the cyclical unemployment as a process that reverts to a zero mean, captures the essence of Friedman's (1968) natural rate hypothesis - that the unemployment rate cannot drift away from the natural rate indefinitely.

Finally, equation (7) specifies the trend output that goes into the Okun equation (2). It follows a random walk process with constant and positive drift.[10] The estimate of the constant drift can be interpreted as the average growth rate of trend output.

Once the model is written in state-space format, and adequate starting values are chosen, its parameters and unobserved variables can be estimated by maximum likelihood using the Kalman filter. Both the state-space form of the model and the Kalman filtering procedure are detailed in the Appendix.

Non-linearity in the measurement system

A major objective of our empirical application is to test for possible non-linearity in the measurement equations (1) and (2). The tests we design are appropriate in a threefold sense: i) model consistent output and unemployment gaps are used; ii) the linear case is nested as a null hypothesis; and iii) convex as well as concave asymmetries are identifiable.

In order to improve on the robustness of the results, three alternative non-linear functional forms are tested for - quadratic,[11] hyperbole, as used by Laxton *et al.* (1995), and exponential. All these forms have the desirable properties of allowing for continuous change in the relevant elasticity across an infinity of possible values, which is an advantage over regime switching or smooth transition models frequently applied in non-linear frameworks. The hyperbole has the additional advantage of modelling explicitly an upper bound for the unemployment gap - the parameter w, below.

The test procedure starts by expressing the relevant unemployment gap elasticity coefficient - γ in the Phillips equation and θ in the Okun's - as a constant plus a function of the last quarter unemployment gap.[12] For the Philips equation (1), the three alternative non-linear forms for γ become (in the Okun equation 2 the expressions for θ are analogous to these)

Quadratic: $\gamma = \gamma_1 + \gamma_2 \times (u_{t-1}^n - u_{t-1})$

Hyperbole: $\gamma = \gamma_1 + \gamma_2 \times \left[\dfrac{w}{w - (u_{t-1}^n - u_{t-1})} \right]$

Exponential: $\gamma = \gamma_1 + \left(\dfrac{1}{u_{t-1}^n - u_{t-1}} \right) \times \gamma_2 \times \left[e^{(u_{t-1}^n - u_{t-1})} - 1 \right]$

The model is then fully estimated with each functional form for each measurement equation, and a test is performed in each of those estimations. The null hypothesis of linearity is $\gamma_2 = 0$, in which case we are back at the basic linear model. If and only if $\gamma_2 = 0$ is rejected, we proceed to estimate the model making sure that the pertinent measurement equation includes the non-linear functional form. In the case of the hyperbole and exponential forms, γ_1 becomes unnecessary and is dropped, once γ_2 is found to be different from zero. As for convexity *versus* concavity of the curves, positive and negative γ_2 indicate, respectively, convexity and concavity

In the next section we apply this procedure to the estimation of the UC model with aggregate Euro Area macroeconomic data.

4. Results

Before reporting and discussing the results of the non-linearity tests for the aggregate Euro Area, we briefly describe the data and some preliminary computations necessary for a correct identification of the model.

Data and preliminary computations

Data are quarterly time series of the GDP deflator, the Imports deflator, the Unemployment Rate and real GDP, for the aggregate of the Euro Area, 1970:I-2001:II. Observations until 1998:III are from the ECB Area Wide Model Database, while those for 1998:IV-2001:II are updates from the ECB Monthly Bulletin.

Unit root tests show that the level of the GDP deflator is integrated of first order, and that the first differences of its log are stationary. This implies that, in Ball's (2000) spirit, near-rational agents form expectations with the best univariate linear time-series model fitted to inflation changes, instead of inflation levels. Standard Box-Jenkins techniques lead to the identification and estimation of the following AR (3) model:

$\Delta\pi_t$ =	$- 0.409\ \Delta\pi_{t-1}$	$- 0.143\ \Delta\pi_{t-2}$	$- 0.214\ \Delta\pi_{t-3}$			
[t-stats]	[-4.63]	[-1.52]	[-2.43]		$R^2 = 0.99$	$Q(26) = 20.7$
(signif)	(0.00)	(0.13)	(0.02)		$\phi(1) = -0.77$	(signif 0.76)

We use these estimates to compute near-rational expectations of inflation changes, which are then fed into the Phillips equation. These expectations are two-step ahead forecasts, because we want to compute $E_t\Delta\pi_{t+1}$ under the assumption that information available at time t reports up to t-1.

Next we report the steps taken in order to identify the best specification of the UC set-up, based on the sign and magnitude of the estimates, individual significance statistics, and normality and auto-correlation analysis of the measurement system residuals.

First, estimating the model with all reasonable combinations of lags of the explanatory variables in the measurement equations, it turns out that only current unemployment gaps are significant, both in the Okun and Phillips equations. The latter result means that there is no evidence of the speed-limit effects discussed since Turner (1995), which is in line with Richardson *et al.* (2000), but not with McAdam and McMorrow (1999).

Second, when trying alternative specifications for the transition dynamics of the NAIRU and trend output, the *a-priori* specifications of equation (4) and (7) above are unquestionably not rejected.

Third, in searching for the most significant variables that might proxy for supply-shocks conditioning the inflation dynamics in the Euro Area, and after testing for several alternative measures of both foreign and domestic shocks, we achieve a parsimonious specification using only the deviation of domestic inflation from the imports deflator inflation, lagged by one quarter.[13] A further advantage of this proxy for the supply shock, lagged once, is that it becomes orthogonal to the near-rational expectations - as imported inflation takes at least one quarter to impact on domestic inflation - and, hence, the generated regressor problem, referred to above, in footnote 5, does not apply to our model.

Fourth, we allow for the presence of *hysteresis*, modelled as a lagged feed-back from the unemployment cycle to the NAIRU in its transition equation, as in Jaeger and Parkinson (1994). This effect is not statistically significant.

Fifth, we check if the fact - referred to above in footnote 5 - that near-rational expectations are not strictly model-consistent could be affecting the results. Specifically, we estimate the model with the AR(3) process for expectations as a fourth measurement equation, and swapping expectations in the Phillips equation by three lags of inflation changes, imposing the cross-equation restriction of equality between these two equations coefficients. As the model hyper-parameters estimates do not change significantly and the unobserved components are observationally equivalent, we proceed with our original near-rational expectations framework.

Finally, we check for evidence in the Euro Area data of any reduction in the weight attached to past inflation in forming inflation expectations, as detected by Brainard and Perry (2000) for the US, and Kichian (2001) for Canada, in the recent years of lower inflation. Specifically, we estimate the model with standard adaptive expectations and let the coefficients on lagged inflation be time-varying. However, its estimates hardly change along the sample and their standard deviations are not statistically different from zero.[14]

Testing for non-linearity

Table 1 summarises the results of the non-linearity tests. For all three functional forms, and irrespectively of modelling the Phillips equation as linear ($\gamma_2 = 0$) or possibly non-linear ($\gamma_2 \neq 0$), the hypothesis that the Okun elasticity is strictly linear is always rejected. Moreover, the sign of the estimates of θ_2 clearly indicate that linearity should be rejected in favour of convex non-linearity in the Okun relation of the Euro Area 1970-2001. This result is in line with the related literature, referred to above.

On the other hand, there is not enough evidence to reject the null of linearity in the Euro Area Phillips relation, at standard confidence levels. This result is somewhat surprising, in view of the theoretical and empirical arguments stating the case for Phillips asymmetry, reviewed above. But it is clear and robust to the three functional forms and to the specifications of the Okun relation.[15]

Having thus concluded that, with this data set, we would model the Philips equation linearly and the Okun equation with convex non-linearity, we set out to choose which particular functional form should the Okun relation take.

Table 2 synthesises information on several statistical criteria relevant for that choice. We select the quadratic function, as it does not record any bad result among these criteria and fares clearly better in three that we consider essential - centricity of the estimated gap near zero, variance of the NAIRU estimation, and normality of the Okun equation residuals.

Trade-off, trend and cycle

Table 3 summarises the results of estimation of the selected model - the UC model presented in section 3, with a quadratic Okun equation instead of a linear one. All the coefficients are significant, and have the expected signs and reasonable estimates.[16]

The Phillips elasticity estimate means that for each additional percentage point of unemployment gap (NAIRU minus actual unemployment rate), inflation changes by an additional 0.043 percentage points per quarter. This elasticity is statistically significant, showing that, with a well specified model, the Phillips relation holds in the Euro Area, contrary to Gali *et al.* (2001) findings. The coefficient on near-rational inflation expectations (α) is not significantly different from 1, as New Keynesian theory predicts, which adds likelihood to this hypothesis of expectations formation.

The Okun coefficients indicate that each additional percentage point of unemployment gap is associated with an additional deviation of output from its trend of 2.3 percentage points plus 2×0.009 of the unemployment gap. Thus, this marginal effect is close to the current textbook benchmark (around 2) for the US Okun law coefficient - Gordon (1998b). It is also close to the estimates by Lee (2000) obtained with annual post-war data of 16 OECD countries.

Although borderline for short-memory auto-correlation, the Phillips equation residuals comfortably pass the standard test for normality, suggesting that the preliminary identification work has been successful. The forecast errors of unemployment also pass the tests, while the one-step-ahead forecast errors of output are not serially correlated, but are on the borderline for rejection of the null of normality (5 percent of significance).[17]

The standard deviations of the innovations to the NAIRU, to its stochastic drift and to the unemployment gap, are statistically significant, indicating a satisfactory estimation of the signal-to-noise ratio. The estimates of the auto-regressive roots of the unemployment gap are consistent with the behaviour generally associated to business cycles - high persistence and hump-shaped response to shocks.

The constant drift of trend output is quite precisely estimated at 0.6 percent, meaning that the average annual growth rate of trend real output is about 2.4 percent, which is in line with conventional wisdom on the matter.

Charts 1-3 show the main unobserved components estimated with our model.[18] Chart 1 shows actual unemployment and the NAIRU. The NAIRU estimates increase systematically until 1994, when they peak at 10.3 percent, and then decrease steadily to a value of 8.7 percent at the end of the sample.

The path of the NAIRU estimated drift, depicted in chart 2, is particularly informative about the path of equilibrium unemployment in the Euro Area during the last three decades. The NAIRU drifted up at a constant rate of about 0.12 percentage points per quarter during the 70s. Then from 1981:II on, the drift rate decreased, implying diminishing increases in the NAIRU. Our model locates in 1994:III the beginning of the recent decrease in the Euro Area NAIRU, and estimates that by the end of the sample the NAIRU drifts down about 0.11 percentage points per quarter. These results are in line with the literature on the European equilibrium unemployment, referred to above, in section 1.

Chart 3 shows the estimates for the unemployment cycle in the Euro Area 1970-2001. They indicate that the Euro Area was in a cyclical trough around the beginning of 1985, and has recovered to reach a cyclical peak by the beginning of 1991. The subsequent deterioration of the cyclical state of the Area has been somewhat interrupted during 1995, but was then resumed, and the Area reached a trough by the end of 1997. Since then, the unemployment gap has been steadily improving and the model identifies a positive gap starting in the second quarter of 2000. In the last quarter of the sample, though, the model already predicts a cyclical downturn in economic activity.

Charts 1 and 3 include 95 percent confidence bands for the NAIRU and the gap, computed with 10000 draws from a Monte Carlo experiment as suggested in Hamilton (1994, pages 397-399). Monte Carlo integration is needed here because the Kalman recursive equations generate a measure of filter uncertainty only, while total uncertainty for the unobserved components includes also parameter uncertainty. As chart 3 shows, it turns out that our model identifies recessions much more precisely than expansions. In fact, we can be 95 percent confident that the economy is in recession about two and a half to three years in advance of the actual trough, while, on the contrary, we can never be that confident that the economy is in expansion.

Table 4 presents further details of the path of the NAIRU and the gap, as well as the uncertainty around both. The 95 percent confidence band varies between 0.71 and 1.78 percentage points throughout the sample, averaging to 1.17 percentage points and amounting to about 1.22 points by the end of the sample. This means that a 95 percent confidence interval is as wide as 2.45 percentage points in 2001:II. These results concerning the imprecision in the estimation of the NAIRU within our UC set-up are coherent with the related literature - see Staiger, Stock and Watson (1997a and b, 2001).

Finally, in table 5 we compare the cyclical turning points estimated with our model with those implicit in the ECB Area Wide Model Database and those given by the inefficiency wedge of Gali *et al.* (2001).[19] There are two major divergences between our results and those of the ECB Gordon-style model. First, the path of the gap during the first seven years of the 70s seems more erratic than ours, and the trough that we date at 1975:IV is dated almost two years later in the AWMD. Second, the cyclical trough in the middle of the 80s is dated in the AWMD one year sooner than in our results.[20]

In table 5, the cyclical turning points identified by Gali *et al.* (2001) inefficiency wedge, for 1980-1998, are highly similar to ours. This result is interesting on two grounds. First, it is remarkable that two such different empirical frameworks (including different estimation methods and statistical information) generate such similar business cycle datings. Second, this result challenges Gali *et al.* (2001) theoretical argument that the assumption of a constant wage mark-up is necessary for the existence of the New Keynesian Phillips curve based on the output/unemployment gap. The bottom half of Gali *et al.* (2001, page 1264) figure 5 shows an apparent change of the wage mark-up during 1980-1998, hence seemingly rejecting their necessary condition. Yet, our research, based on unemployment and output gaps and forward-looking near-rational expectations of inflation, estimates a significant Phillips relation and generates a very similar business cycle dating for the Euro Area.

5. Concluding remarks

The objective of this paper has been to address the trade-off between the unemployment gap and inflation changes in the Euro Area 1970-2001, with an emphasis on the possibility of non-linearity. The estimated trivariate unobserved components model seems to capture reasonably well features of the macroeconomic data of the aggregate Euro Area during this period, and clearly identifies a statistically significant Phillips trade-off. The peaks and troughs of the estimated unemployment gap seem in accordance with the conventional wisdom about recent European cycles and are very close to turning points identified by alternative sources with different frameworks, data and method.

The design of our model-consistent tests for non-linearity always nests the null hypothesis of linearity, and includes the choice of several non-linear functional forms. Somewhat surprisingly, in view of recent literature on this subject, the Phillips curve turns out to be linear, but non-linearity clearly shows up in the Okun relation. We draw two immediate consequences from these findings. First, the choice of unemployment *vs* output gap to characterise the trade-off is not irrelevant in terms of its linearity/non-linearity. Second, neither non-linearity nor linearity should be taken for granted *a priori* across samples.

We have used in the Phillips equation a concept of forward-looking near-rational expectations, which, in our view, improves on the contradiction between rational expectations and evidence of inflation inertia. The unobserved components model, applied to the Euro Area data, seems to have reacted very well to the inclusion of this type of expectations. Clearly, being at the heart of the unemployment-inflation trade-off, further knowledge about expectations formation is essential to current research of the type carried out in this paper.

As for the precision of the trend-cycle decompositions, the confidence bands computed here reveal a great amount of uncertainty, once both filter and parameter variances are properly accounted for. Hence, in the Euro Area, as in other developed countries, the evidence so far calls for a cautious use of the unemployment and output gaps in the practical conduct of monetary policy.

Appendix: State-space form and estimation procedure

The system is written in state-space form as follows:

$$O_t = \alpha t U_t + \varepsilon_t^O \tag{A1}$$

$$U_t = TU_{t-1} + gt + \varepsilon_t^U \tag{A2}$$

where O stands for observables and U for unobservables, and

$$O_t = \begin{bmatrix} y_t \\ u_t \\ \Delta\pi_t \\ \end{bmatrix} \quad \alpha\tau = \begin{bmatrix} 0 & 0 & 1 & 0 & \theta & 0 & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 & 0 \\ Et\Delta\pi(t+1) & St & 0 & 0 & \gamma & 0 & 0 \end{bmatrix}$$

$$\varepsilon_t^O = \begin{bmatrix} \varepsilon^{ok} \\ 0 \\ \varepsilon^{phi} \end{bmatrix} \quad U_t = \begin{bmatrix} c_{1t} \\ c_{2t} \\ y_t^p \\ u_t^n \\ (u_t^n - u_t) \\ (u_{t-1}^n - u_{t-1}) \\ \mu_t \end{bmatrix} \quad g_t = \begin{bmatrix} 0 \\ 0 \\ g \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & \rho 1 & \rho 2 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \varepsilon_t^U = \begin{bmatrix} \varepsilon_t^{c_1} \\ \varepsilon_t^{c_2} \\ \varepsilon_t^{p} \\ \varepsilon_t^{n} \\ \varepsilon_t^{gap} \\ 0 \\ \varepsilon_t^{\mu} \\ \end{bmatrix}$$

All innovations are assumed to be independent, serially uncorrelated and with normal distribution with zero expected values and constant variances.

In this model, α_t is a vector composed of observed variables (expected inflation changes and the supply-shock proxy), zeros, ones and two parameters that are to be estimated - the Phillips slope parameter, γ, and the Okun law parameter, θ.

U_t is a vector of unobserved variables, composed of the coefficients associated to the expectations of inflation and the supply-shock proxy, c_{1t} and c_{2t}, and of the unobservable variables that are to be estimated – trend output, equilibrium unemployment, gap and NAIRU drift. It has transition equations defined by the matrix T. In the transition system, ε_t^U is a vector of innovations, assumed to be normally distributed with a variance-covariance matrix Q:

$$\varepsilon_t^U \sim \mathrm{N}(0,Q) \tag{A3}$$

This matrix Q has, by assumption, all elements equal to zero except the diagonal ones associated with trend output, the NAIRU, the gap and the NAIRU drift - σ_{ε^p}, σ_{ε^n}, $\sigma_{\varepsilon^{gap}}$ and σ_{ε^μ} - so that these are the only parameters that really vary with time.

Estimation by the Kalman recursive equations requires the setting of initial values for the state vector. The starting values of trend output and the NAIRU have been set to the corresponding observed values of output and unemployment, and those of the gap and drift were set accordingly to that assumption. Following the suggestion in the literature, relatively diffuse priors were adopted here by assuming large starting values for the unobserved variance matrix. Specifically, the standard deviation of the one-step ahead predictions of the system unobserved components were initialised at 5 percent for trend real output, 2 percentage points for the NAIRU and gap and at 0.75 percentage points for the NAIRU drift.

With these starting values for the state system and with initial conditions for the likelihood hyper-parameters, the Kalman iterations are run aiming at the maximisation of the likelihood function. The log-likelihood function, L, is written on the system one-step ahead prediction errors and their variances, as described in Harvey (1989, pages 125-128). In the case of our baseline linear model L is a function of eleven unknown parameters, sometimes called hyper-parameters,

$$L = L(\gamma, \theta, \rho 1, \rho 2, g, \sigma^{ok}, \sigma^{phi}, \sigma^{n}, \sigma^{p}, \sigma^{gap}, \sigma^{\mu}) \tag{A4}$$

which are, respectively, the slope parameters in the Phillips and Okun equations, the two auto-regressive roots in the unemployment gap process, the average rate of growth of trend output, the standard deviations for the disturbances in the Okun and Phillips equations, and the standard deviations for the innovations in the NAIRU, the unemployment gap, trend output and the NAIRU drift.

Estimation is carried out in *GAUSS*, using the procedure *Optmum* to maximize the likelihood function. Iterations have been typically initiated with the *BFGS* algorithm and the *stepbt* step-method, and then switched to the *Newton* algorithm and the *half* step-method when the function had come closer to the maximum. Standard deviations of the hyper-parameters estimates were computed from the inverse of the Hessian, while variances of the time-varying parameters were obtained from the Kalman filter recursive equations.

Notes

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[1] Laxton *et al.* (1995), Clark *et al.* (1995, 1996), Clark and Laxton (1997), Laxton *et al.* (1999), and Isard *et al.* (2001) present these arguments in terms of deterministic and stochastic *NAIRUs*, not *gaps*. This is because they model the asymmetry as dependent on the *level* of the rate of unemployment, rather than on the unemployment *gap*. We, instead, argue in terms of unemployment *gap*, which is the appropriate concept in a model with a time-varying NAIRU.

[2] A recent result that may corroborate this point has been offered by Hamilton (2001). Applying his parametric method of flexible non-linear inference to US annual inflation and unemployment data from 1949 to 1997, he found no evidence of non-linearity in the Phillips function with the unemployment *level*.

[3] An alternative solution is Stock and Watson (1998) median unbiased estimator, used in Gordon (1998a) and Staiger, Stock and Watson (2001).

[4] For a simple deduction of the Calvo-Rotemberg Phillips equation see, *inter alia*, Roberts (1995).

[5] Two estimation problems could arise from this procedure. One derives from the limited information nature of the near-rational expectations, which may not be strictly model-consistent. And the other has to do with a possible generated regressor problem, as discussed in Pagan (1984) and Pagan and Ullah (1988), caused by the use of the ARIMA generated series of expectations as an explanatory variable in the Phillips equation. In section 4, below, we empirically check on both these potential problems.

[6] A possible alternative would be to model hybrid backward and forward-looking expectations as suggested by Fuhrer (1997) and subsequently used by, *inter alia*, Rotemberg and Woodford (1997), Gali and Gertler (1999), and Roberts (1998, 2001). But this solution is

Notes (continued)

theoretically *ad-hoc* and empirically controversial with respect to the weight attached to each side of expectations.

[7] Equations (1) and (2) are presented in their baseline linear specification. The non-linear alternative functional forms are described below.

[8] Without the control for temporary supply shocks, the estimates would really refer to a short-run NAIRU - the level of unemployment consistent with maintaining the current level of inflation in next period. For details on the distinction between the short-run NAIRU, the (stochastic) NAIRU and the natural rate (or long-term/deterministic NAIRU), see, *inter alia*, Estrella and Mishkin (1998) and Richardson *et al.* (2000).

[9] This has been used, *inter alia*, by Laubach (2001) and Kichian (1999).

[10] The different times-series specifications of the unemployment and output trends - equations (4) and (7) - are suggested by the behaviour of the observed series. This is intentional, as we, contrary to other researchers of this subject, do not regard as necessary any further restrictions. For example, Camba-Méndez and Rodriguez-Palenzuela (2001) and Fabiani and Mestre (2001) impose that both trends follow the same dynamic process.

[11] The use of the quadratic form in this context has been first suggested to us by St. Aubyn (2000).

[12] The non-linearity is conditional on last quarter gap, which means that this model falls into the category of Conditionally Gaussian Models described in Harvey (1989, pages 156-160), to which the Kalman filter can be optimally applied.

[13] Measures of domestic shocks tried here include several different productivity series. Alternative measures of foreign shocks have been tried especially because the imports deflator in the AWMD was built by aggregation of the national imports deflators, thus including changes in prices of trade between the member-states - see Fagan *et al.* (2001). In the end, however, the domestic deviation from imported inflation is the most significant variable.

[14] Alternatively, we have also tried letting the coefficient on the near-rational expectations of inflation in the Phillips equation be a time-varying parameter - a test closer to Akerlof *et al.* (2000) - but still found no evidence of significant change in this parameter.

[15] The test of the hyperbolic functional form follows a sequential procedure. First, the wall parameter (w) is estimated in a strictly hyperbolic model, that is, not including any linear

Notes (continued)

component in the relevant elasticity. Then, this wall estimates are calibrated into the model with both linear and hyperbolic components in the relevant elasticity, in order to assess the significance of γ_2 (or θ_2). The specific wall parameters values used in the tests are 2.2 for the Okun equation and 1.1 for the Phillips. For sensitivity analysis we also checked that the results were unchanged with w equal to 2 and to 3. Recall that w means the unemployment gap at which the output gap (in the Okun curve) or inflation changes (in the Phillips equation) would increase without bound.

[16] The standard deviation of the innovation to the Okun equation residuals converges systematically to 0, both in the linear and non-linear specifications, and, hence, is restricted to that value.

[17] Note, however, that the result is far worse when we estimate a linear Okun equation.

[18] The reported values are those given by the Kalman smoother, which uses the information of the whole sample, as described in Harvey (1989, pages 154-155).

[19] See how Ross and Ubide (2001) compare the cyclical turning points in the Euro Area arising from several alternative trend-cycle decomposition methods.

[20] The failure of the ECB AWMD in dating the 1997 trough is probably not relevant, since their model was estimated with data ending in 1998:III.

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Table 1. Non-linearity tests

	Sign of estimate	Significance	Inference
Okun non-linearity			
Quadratic			
H0: $\theta 2=0$			
given: $\gamma 2=0$	+	0.01	Rejected
$\gamma 2\neq 0$	+	0.01	Rejected
Hyperbole			
H0: $\theta 2=0$			
given: $\gamma 2=0$	+	0.00	Rejected
$\gamma 2\neq 0$	+	0.00	Rejected
Exponential			
H0: $\theta 2=0$			
given: $\gamma 2=0$	+	0.00	Rejected
$\gamma 2\neq 0$	+	0.00	Rejected
Phillips non-linearity			
Quadratic			
H0: $\gamma 2=0$			
given: $\theta 2=0$	-	0.88	Not rejected
$\theta 2\neq 0$	+	0.34	Not rejected
Hyperbole			
H0: $\gamma 2=0$			
given: $\theta 2=0$	+	0.52	Not rejected
$\theta 2\neq 0$	+	0.58	Not rejected
Exponential			
H0: $\gamma 2=0$			
given: $\theta 2=0$	-	0.44	Not rejected
$\theta 2\neq 0$	+	0.22	Not rejected

Table 2. Selection of non-linear functional form

		Quadratic	Hyperbole	Exponential
Criteria				
Significance $\theta 2$		0.0106	0.0000	0.0000
Significance γ		0.0364	0.2821	0.0273
Variance estimation NAIRU		0.2844	0.5790	0.3409
Variance estim. Trend GDP		0.0002	0.0004	0.0003
MSE $\Delta\pi$		0.0975	0.0978	0.0975
U GAP:	mean	-0.0532	-1.0436	-0.8248
	min	-1.1835	-2.8421	-2.3398
	max	1.7533	1.0532	0.9971
Normality residuals:				
	Okun	0.0528	0.0065	0.0131
	Unemployment	0.1242	0.1043	0.0117
	Phillips	0.6461	0.1833	0.1470

Note: The normality residuals results are the significance probability of the Jarque-Bera statistic under the null hypothesis of normality.

Table 3. Model estimation results

	Estimate	T-Statistic	Significance
Phillips equation			
γ	0.043	2.09	0.04
α	1.020	0.11	0.92 *
ω	0.050	4.14	0.00
σ ε(phi)	0.284	15.37	0.00
OKun equation			
$\theta 1$	0.023	4.54	0.00
$\theta 2$	0.009	2.56	0.01
Unobserved Comp.			
Trend GDP			
g	0.006	13.09	0.00
σ ε(p)	0.005	13.38	0.00
NAIRU			
σ ε(N)	0.093	8.11	0.00
σ $\varepsilon(\mu)$	0.017	2.07	0.04
U Gap			
$\rho 1$	1.800	23.75	0.00
$\rho 2$	-0.824	-11.17	0.00
σ ε(c)	0.060	4.64	0.00
Residuals			
Phillips equation			
sample mean	-0.035		0.70
Jarque-Bera	0.874		0.65
Q(4)	9.350		0.05
Q(24)	30.036		0.18
Okun equation			
Jarque-Bera	5.883		0.05
Q(4)	1.696		0.79
Unemployment eq.			
Jarque-Bera	4.172		0.12
Q(4)	5.641		0.23

Notes: α is the coefficient associated with expectations of inflation changes.
* significance probability under the null hypothesis $\alpha = 1$.
The normality residuals results are the significance probability of the Jarque-Bera statistic under the null hypothesis of normality.

Chart 1. NAIRU, confidence bands, and unemployment

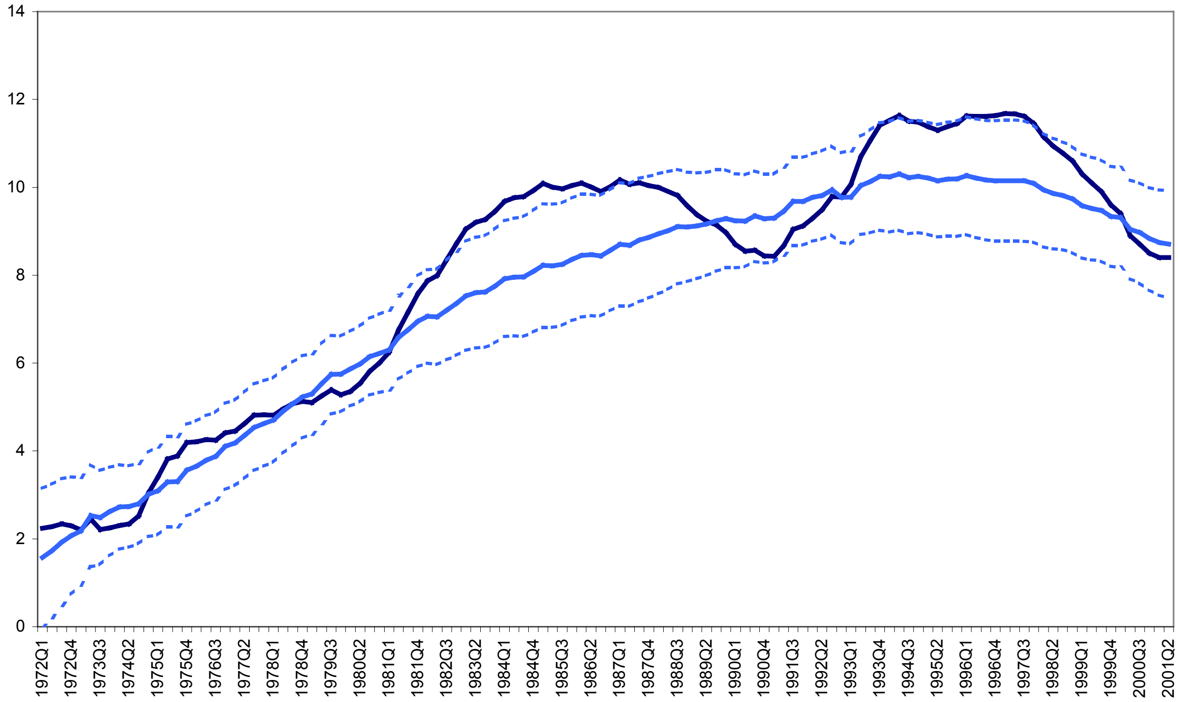

Chart 2. NAIRU drift

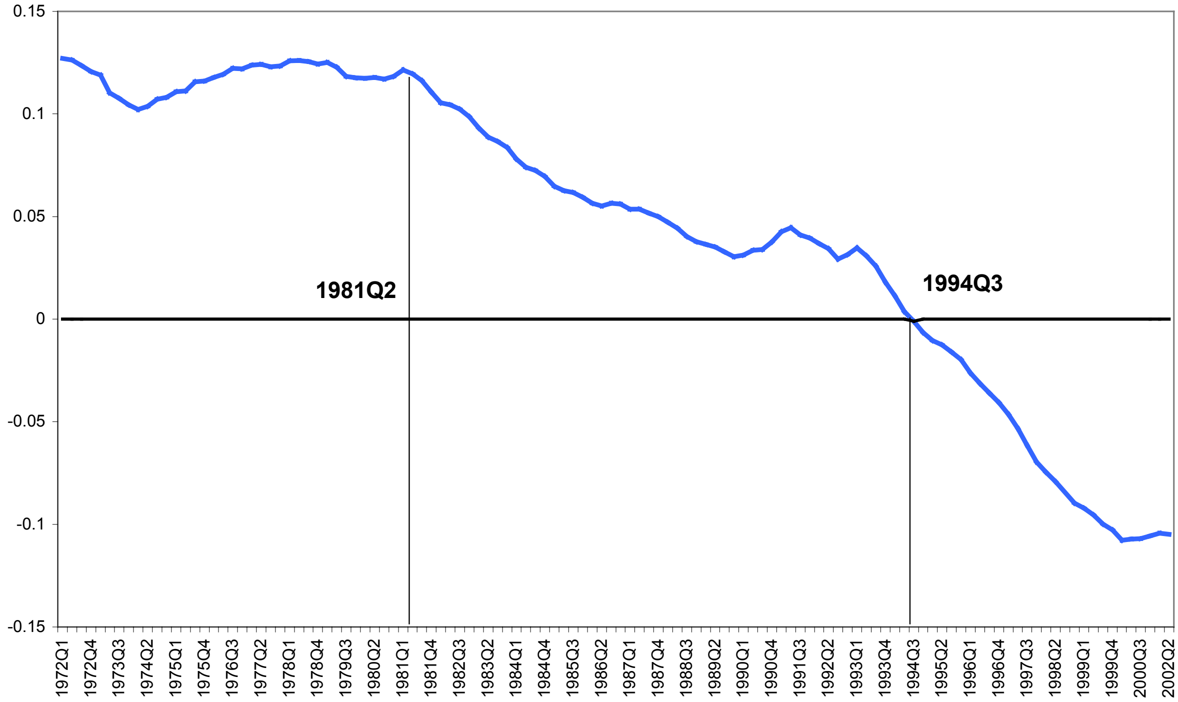

Chart 3. Unemployment gap and confidence bands

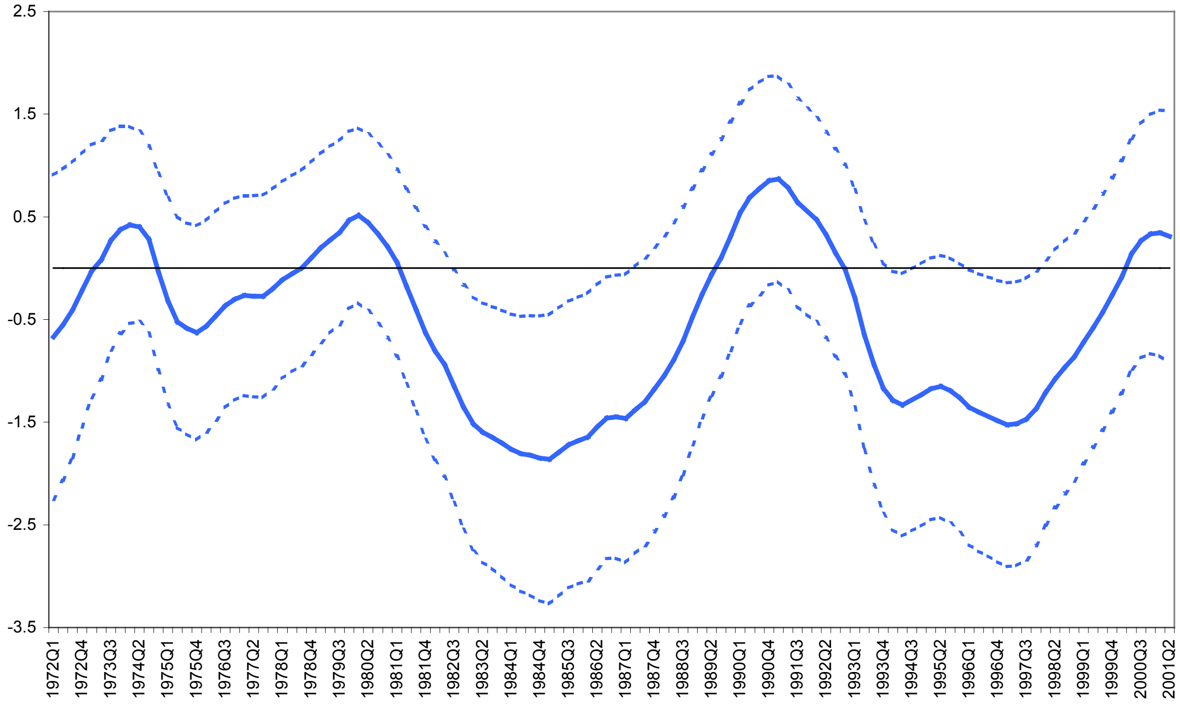

Table 4. NAIRU and gap - smoothed estimates and uncertainty

	2001:II	1972:I-2001:II		
		Average	Maximum	Miminum
NAIRU	8.71	7.48	10.31	1.57
Unemployment GAP	0.31	-0.55	0.87	-1.86
MSE	0.39	0.36	0.83	0.13
Filter	0.25	0.14	0.25	0.07
Parameter	0.14	0.22	0.58	0.06
RMSE (standard error)	0.62	0.60	0.91	0.36
95% confidence band (1.96 * RMSE)	1.22	1.17	1.78	0.71
95% confidence interval (± 1.96 * RMSE)	2.45	2.35	3.57	1.42

Table 5. Cyclical turning points

	Our results *trivariate UC*	Gali, Gertler & Salido (2001) *Inefficiency wedge*	ECB AWMD (2001) *bivariate UC*
Peak	1974:I	-----	1974:II
Trough	1975:IV	-----	1977:III
Peak	1980:I	1980:IV	1980:I
Trough	1985:I	1984:IV	1984:I
Peak	1991:I	1991:I	1990:IV
Trough	1997:I	1997:III	1994:II