Optimizing a medical image registration algorithm based on ¹ profiling data for real-time performance

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Carlos A. S. J. Gulo *·* Antonio C. Sementille *·* ³ João Manuel R. S. Tavares ⁴

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Abstract Image registration is a commonly task in medical image analysis. \overline{z} Therefore, a significant number of algorithms have been developed to perform rigid $\frac{1}{8}$ and non-rigid image registration. Particularly, the free-form deformation algorithm 9 is frequently used to carry out non-rigid registration task; however, it is a 10 computationally very intensive algorithm. In this work, we describe an approach 11 based on profiling data to identify potential parts of this algorithm for which parallel $_{12}$ implementations can be developed. The proposed approach assesses the efficient of 13 the algorithm by applying performance analysis techniques commonly available in $_{14}$ traditional computer operating systems. Hence, this article provides guidelines to ¹⁵ support researchers working on medical image processing and analysis to achieve 16 real-time non-rigid image registration applications using common computing 17 systems. According to our experimental findings, significant speedups can be 18 accomplished by parallelizing sequential snippets, i.e., code regions that are ¹⁹ executed more than once. For the selected costly functions previously identified in $_{20}$ the studied free-form deformation algorithm, the developed parallelization 21 decreased the runtime by up to seven times relatively to the related single thread 22 based implementation. The implementations were developed based on the Open ²³

Carlos A. S. J. Gulo

CNPq National Scientific and Technological Development Council

Research Group PIXEL - UNEMAT, Brazil

Programa Doutoral em Engenharia Informática, Instituto de Ciência e Inovação em Engenharia Mecânica e Engenharia Industrial, Faculdade de Engenharia, Universidade do Porto, Portugal E-mail: sander@unemat.br

Antonio C. Sementille

Departamento de Ciências da Computação, Faculdade de Ciências, Universidade Estadual Paulista-UNESP, Brazil

E-mail: antonio.sementille@unesp.br

João Manuel R. S. Tavares

Instituto de Ciência e Inovação em Engenharia Mecânica e Engenharia Industrial, Departamento de Engenharia Mecânica, Faculdade de Engenharia, Universidade do Porto, Portugal E-mail: tavares@fe.up.pt (corresponding author)

 Multi-Processing application programming interface. In conclusion, this study confirms that based on the call graph visualization and detected performance

bottlenecks, one can easily find and evaluate snippets which are potential

optimization targets in addition to throughput in memory accesses.

Keywords Medical image processing and analysis *·* Profiling tools *·* Performance

analysis · Non-rigid image registration

30 1 Introduction

31 Medical image analysis plays a significant role in the field of medicine, and image registration is an important and widely used technique in this context. Today, patients are imaged on routine basis using different imaging systems. Patients are also monitored over time to assess disease progression or response to therapy. However, to be able to study physiological and/or structural changes over time, or to combine complementary information that different imaging systems produce, it is necessary to perform the registration of the acquired images [1]. Image registration is a computational task that determines the spatial correspondence between two images of the same object acquired at different angles, at different times, using different image modalities, or under different acquisition conditions [2, 3, 4]. In general, an image registration method can be decomposed into three parts: building a transformation model, computing a similarity measure and performing the 43 optimization of the registration model [4, 5]. Transformation models, such [as](#page-16-0) [rig](#page-16-1)id or non-rigid models, delineate the transformation that can be used to represent the underlying correspondences. Rigid models describe simple linear mappings such as translations, rotations, scalings and shears. However, non-rigid transformation models can represent more complex mappings, since local deformations are also taken into account, resulting thus in longer computation times [6, 5]. Non-rigid image registration is an extensive research field, encompassing many applications and several specific algorithms. For example, the ones based on mutual $_{51}$ informa[t](#page-17-0)ion [7, 8], elastic tr[ans](#page-17-1)formation models [9], multi-resolution [10], and similarity measures [6]. However, the r[equ](#page-17-2)ired computational effort is frequently high when a non-rigid image registration algorithm is used. Hence, this task is well-known as one of the most time-consuming tasks that can be found in medical image analysis [11, 12]. How[ever,](#page-17-3) [with](#page-17-4) the development of multi-core processor architecture, several solutions have been proposed that realize non-rigid image registration algorithms on multi-core CPUs [13, 14, 15]. Multi-core architecture mainly aim at improving the performance of highly demanded applications by exploiting parallelism. However, writing parallel algorithms from scratch is a very complex and demanding task. Furthermore, parallelizing legacy algorithms is even more challenging [16, 17, 18]. Fort[unate](#page-17-5)[ly,](#page-17-6) a [profi](#page-17-7)ling method can be effectively used to identify and evaluate portions of code responsible for consuming excessive computational resources $[17, 18]$. For example, a [profil](#page-17-6)[ing](#page-17-7) tool can accurately count the activation instances of a function during runtime of an algorithm. Furthermore, it ⁶⁵ can provide timing information about the function [19]. Profiling is therefore a helpful approach in program optimization, which is based on gathering and

In the presented study, we employed profiling tools to identify functions with τ long run-times in a popular image registration algorithm: the Free-Form π Deformation (FFD) algorithm $[12, 11]$. Based on the collec[ted](#page-17-4) [profili](#page-17-3)ng data, we $\frac{74}{4}$ carried out a performance analysis of the algorithm. In particular, we aimed to $\frac{75}{15}$ effectively decrease its processing time in order to adapt it to be feasible for π real-time diagnosis. To this end, we exploited computational resources typically τ available in modern personal computers. We gauged "performance" by working out 78 the operating systems efficiency during algorithms execution. This evaluation took τ into account the factors of throughput, latency, and availability. Thus, throughout 80 this article, we provide guidelines and methods that can support researchers of ϵ ⁸¹ medical image processing and analysis in identifying very time consuming functions ⁸² in their algorithms using profiling tools. The experimental findings show that the \approx gathered profiling information can point out the main bottlenecks found in an $\frac{84}{100}$ algorithm implemented in C. This study also provides insights into why profiling $\frac{1}{100}$ data is useful; in particular, for optimizing a non-rigid image registration algorithm 86 for real-time applications.

To the best of our knowledge, this is the first time that the chosen profiling tools $\frac{88}{100}$ are used to support the parallelization of a non-rigid image registration algorithm. $\frac{1}{80}$ Our findings are thus highly pertinent for the image processing and analysis area, ₉₀ mainly for the medical imaging community. Frequently, medical images in real θ clinical scenarios are of high resolution and need to be processed and analyzed fast. ⁹² Additionally, computers with multi-cores are available in medical environments 93 with enough computational power to handle tasks of image processing and analysis θ efficiently. Hence, the insights presented in this work are timely and demanded for ϵ researchers developing algorithms of medical image processing and analysis.

This article is organized as follows: Section 2 presents the background concepts, [97](#page-3-0) mainly the profiling method used to identify snippets with excessive CPU 98 consumption. In the same section, methods that have been proposed to speedup the $\frac{99}{96}$ computation of non-rigid image registration algorithms are reviewed. In Section 3, ¹⁰⁰ the material and methods used to speedup the runtime of the studied algorithm of $_{101}$ non-rigid image registration, including the profiling tools used for tasks such as ¹⁰² measuring the performance of the algorithm, gathering the data to be analyzed, and 103 building the visualization of the performance analysis, are described. The main ¹⁰⁴ findings and observations resulting from the performed experiences using profiling 105 data to optimize the computation of the studied algorithm are discussed in Section 4. ¹⁰⁶ Section 5 provi[des](#page-15-0) the conclusion of this study and presents future work directions. 107

2 Background and Related Work

 In this section, the topic of medical image registration and the profiling tools used in this study are introduced. Additionally, research concerning the use of high-performance computing techniques to speedup medical image registration algorithms is reviewed.

2.1 Medical image registration

 Image registration is the process of aligning images of the same object obtained at different times or from different view-points, using different or similar imaging modalities or conditio[ns](#page-18-3) $[16, 25, 8]$. This process [alig](#page-17-5)ns [geom](#page-17-1)etrically two images, usually referred as the *reference* and *sensed* images. In image registration applications, the involved information can be gathered through a combination of data sources as in image fusion, change detection, and multi-channel image 120 restoration, to name a few $[14, 26]$. Focusing on no[n-rigid](#page-17-8) [reg](#page-18-4)istration, one accounts for changes between the images that arise not only by global rotations, translations and scaling, but also due to complex local variations. Medical image registration is commonly used to follow up information on patient anatomy along different time points, where one must take into account the deformation of the anatomy itself due to, for example, the patient's breathing or normal anatomical changes [9, 14].

 A significant number of image registration methods have been developed both to obtain the combination, i.e., fusion of data acquired by different clinically useful imaging modalities through mutual co-registration, or to register one image to other images to understand how patient anatomy has changed over time $[14, 15]$. In general, the majority of the rigid image registration methods comprise four steps: feature detection, feature matching, transform model estimation, and image re-sampling and transformation [25, 14]. Non-rigid registration [me](#page-18-3)[thods](#page-17-8), on the other hand, commonly search for the optimal transformation parameters that maximise a similarity measure. All these steps are well documented in literature [25, 14, 27, [15\]](#page-18-3)[.](#page-17-8)

 In order to attained the registration of two input images, the non-rigid registration should establish a correspondence measure between a reference image, 138 *I_r*, and sensed image, *I_s*, using a parameter transformation $T_t(\cdot)$ of image geometry 139 in line with a similarity function $\rho(\cdot)$. When I_s has a higher dimension than I_r , 140 projection operators P_r and P_s can be used to reduce I_s dimensionality. Then, the non-rigid image registration problem can be expressed via maximizing the similarity $_{142}$ measure function [26]:

$$
T_t^*(\cdot) = \arg_{T_t(\cdot)} \max \rho(P_r(I_r), P_s(T_t(I_s))). \tag{1}
$$

 An FFD model comprises a powerful tool for deforming an image volume using cubic B-splines. This technique is applied, for example, in deformation analysis in brain images, by deforming an object by adjusting an underlying mesh of control points, creating its 3D shape, and a smooth and C^2 continuous transformation [12]. To define a spline based FFD, the domain of the image volume can be denoted as

 $Q = \{(x, y, z) | 0 \le x < X, 0 \le y < Y, 0 \le z < Z\}$. On the other hand, let the 148 parameters of the transformation and the amount of deformation. Φ , be expressed as parameters of the transformation and the amount of deformation, Φ , be expressed as a $n_x \times n_y \times n_z$ mesh of $\phi_{i,j,k}$ control points with a uniform spacing, δ . Thus, ϕ_{150} can be formed as a low resolution mesh for modeling global non-rigid deformations, ¹⁵¹ and as a high resolution mesh for modeling local deformations of the control points ¹⁵² mesh $[11, 12]$ [with](#page-17-3) [hig](#page-17-4)h accuracy. Thus, one can write FFD as a 3D tensor product of $\frac{153}{153}$ 1D cubic B-splines expressed as: ¹⁵⁴

$$
T_{local(x,y,z)} = \sum_{l=0}^{3} \sum_{m=0}^{3} \sum_{n=0}^{3} B_l(u) B_m(v) B_n(w) \phi_{i+l,j+m,k+n}, \tag{2}
$$

where $i = \lfloor x/n_x \rfloor - 1$, $j = \lfloor y/n_y \rfloor - 1$, $k = \lfloor z/n_z \rfloor - 1$, $u = x/n_x - \lfloor x/n_x \rfloor$, 155
 $v = y/n_x - \lfloor y/n_x \rfloor$ and $w = z/n_x - \lfloor z/n_x \rfloor$ and B_t represents the *L*th basis $v = y/n_y - |y/n_y|$, and $w = z/n_z - |z/n_z|$, and B_l represents the *l*-th basis function of the B-spline $[11, 12]$: 157

$$
B_0(u) = (1 - u)^3/6,
$$

\n
$$
B_1(u) = (3u^3 - 6u^2 + 4)/6,
$$

\n
$$
B_2(u) = (-3u^3 + 3u^2 + 3u + 1/6,
$$

\n
$$
B_3(u) = u^3/6.
$$
\n(3)

Considering $B_l(u) = 0$ for $l < 0$ and $l > 3$, the derivative terms are nonzero 158 only in the neighborhood of a given point. Therefore, the optimization of the ¹⁵⁹ objective function can be efficiently achieved using gradient descent $[11, 12]$. However, FFD algorithm is computationally intensive; in particular, when dealing 161 with images of large dimensions, which occurs frequently in several possible 162 applications $[14]$. As an [exam](#page-17-8)ple, the parallel computation of the human brain 163 deformation is a recent field of exploration which can be efficiently studied through ¹⁶⁴ processing large amounts of high resolution images concurrently $[3]$. Moreover, the 165 conjugate gradient descent algorithm can optimize all control points and interpolate ¹⁶⁶ the complete image under study at each iteration $[11]$. However, the computation of 167 the similarity measure and of the geometric transformation are computational 168 bottlenecks of the non-rigid registration method, demanding, therefore, increased ¹⁶⁹ efforts for effective parallelization techniques for such computations. ¹⁷⁰

2.2 Profiling methods 171

Here, the use of profiling methods for measuring the computation time of each 172 function in an algorithm is described. Profiling is a well-known tool that evaluates 173 the performance of an algorithm by gathering its runtime data. Its principal objective 174 is to assist programmers in identifying performance bottlenecks of the algorithm. 175 Thus, the technique is commonly used to get an insight into an algorithm's ¹⁷⁶ performance, and to assess the use of instruction sets in order to identify and ¹⁷⁷ evaluate portions of code that cause excessive processor utilization. It can further 178 check several metrics such as memory allocation, memory usage and leaks, cache ¹⁷⁹ ¹⁸⁰ performance, execution time, or even energy consumption [22]. Different profiling

 approaches exist such as instrumented, event based, statistical, and simulation based $[20, 19, 18]$.

Usually, a performance analysis based on profiling consist of the following four

steps: instrumentation or modification of the algorithm under study to generate

performance data, measurement of significant aspects of execution that are essential

 for generating the needed data, analysis and visualization of the gathered data [24] (Fig. 1).

Fig. 1 Diagram of the profiling method.

2.2.1 Instrumentation

 A compiler and the source code of the algorithm under study are needed for its instrumentation. The instrumentation process, at compile time, adds a detailed listing of the running statistics to the object file, and links the executable to standard libraries that have profiling information enabled. Next, the instrumentation incorporates measurement code into the implementation, resulting in an accurate assessment of running times [17, 28, 24]. Instrumentation [pr](#page-17-6)[ocess](#page-18-5)[es](#page-18-6) are developed to determine possible options for modifying the behavior of the algorithm. Monitoring runtime behavior of algorithms involves aggregating information based on the number of executions of each basic block, and instrumenting binaries to trace various type of events such as free and malloc and similar function utilities.

2.2.2 Measuring

Gathering profile data is the second step in the profiling method. Typically, the

- following information is collected during the execution of the algorithm under
- study: approximate time spent in each function; the number of times a function is invoked; a list of the caller functions invoking a given function; a list of the

descendant functions that a given function invokes; and an estimate of the ²⁰⁴ cumulative time spent in the descendant functions invoked by a given function $[19]$. Relevant data about functions can be collected by post-processing this information ²⁰⁶ from one or more executions. This data is then stored in files for subsequent $_{207}$ processing. Following this, a dynamic call graph for the execution is built $[18, 23]$. 208 In general, gathering profiling data does not interfere with the execution of the 200 $algorithm [23, 24].$ 210

2.2.3 Data analysis 211

The third step of the profiling method concerns the analysis of the data gathered in $_{212}$ the previous step. Here, related binary is produced and, subsequently, the output data 213 is available for extraction. By convention, output files are named after the respective $_{214}$ profiler, e.g., perf.data for the perf profiler, gmon.out for gprof. Each 215 output file contains the execution profile. Profilers analyse the data and extract ²¹⁶ performance statistics as well as record the arc in the call graph that corresponds to ²¹⁷ the activation of each function $[20, 19, 24]$.

Profiler determines the most costly functions and collects the arcs of the dynamic 219 call graph traversed during the execution of the algorithm for such functions. This $_{220}$ enables the visualization of the call graph and presentation of the measures collected ₂₂₁ from the execution. Such information can be graphically represented as returning 222 address for a function call referred as *caller* being used to identify the source of the 223 arc and the destination referred as *callee* [23].

2.2.4 Visualization ²²⁵

In the final step, collected profiling data is presented by incorporating the call graph $_{226}$ of the algorithm under study. Visualization can employ *call stack walking* which is a ²²⁷ technique that identifies calling relationships between functions in an ²²⁸ implementation. Every call relationship that occurs is also represented in the graph $_{229}$ with the CPU usage time for the respective call. Both tools, gprof and perf, 230 provide dynamic call graph information for all instrumented code snippets. A call ²³¹ graph is binary, and sometimes is treated as a multi-graph, instead of as $_{232}$ relations-relation over functions, or procedures as defined in an algorithm 233 implementation [20, 19]. An edge (f, g) (f, g) (f, g) [sh](#page-18-0)ows that function *f* invokes function *g*, 234 and the nodes represent the individual functions in the executable. 235

2.3 Related work 236

Parallel computing has been applied to highly complex problems such as 237 computations that involve large workloads and data, and intensive critical numerical 238 analysis. Sequential algorithm implementations are frequently re-coded in order to ²³⁹ decompose the algorithms or the data into smaller portions. These portions are ²⁴⁰ commonly referred as tasks, and are distributed to be executed in many- or ²⁴¹ 242 multi-cores simultaneously $[29, 30]$. Throughout this [pro](#page-18-7)[cedure](#page-18-8), the tasks of

 communication and coordination are performed based on memory usage by 244 different computer processing units [30].

 The growing popularity and use of multi-core processor architectures in medical imaging applications is well documented [14, 27, 15]. The primary objective o[f](#page-17-8) multi-core CPUs architectures has been to increase the performance of applications by exploiting parallelism. However, writing parallel implementations from scratch is a very complex and demanding task. Parallelizing legacy implementations written by someone else is even more challenging [14].

 From several works [31, 32, 26], one can [re](#page-18-9)[alize](#page-18-10) [that](#page-18-4) in the field of medical image processing and analysis, it is not common for the developers to use tools that detect computationally costly functions in their algorithms. However, several studies have been conducted to address performance issues in image registration algorithms using high-performance computing [33, 13, 15]. For example, Shack[leford](#page-18-11) [et](#page-17-9) [al.](#page-17-10) [\[14\]](#page-17-8) presented a comprehensive survey of non-rigid registration algorithms that are suitable for use in modern multi-core architectures. Image registration tasks though computationally costly can yield to high parallelism. Therefore, multi-core architectures with high parallel processing power provide excellent opportunities for speeding up these tasks.

 Computationally intensive Mutual Information (MI) based algorithms have been successfully employed in parallel architectures such as clusters [34], Graphic Processing Unit (GPU) [33, 27], multi-core C[ell](#page-18-11) [Broadb](#page-18-12)and Engine Architecture (CBEA) [35], and [Field](#page-18-13)-Programmable Gate Array (FPGA) [7], reducing their runtime and making them suitable for routine clinical use. For example, MI based algorithms have been used to correct the misalignment - an image registration application - of tissue in computed tomography (CT), positron emission tomography (PET) and magnetic resonance (MR) images, achieving accuracy comparable to one achieved by clinical experts.

 Rohlfing and Maurer [3] and Christen[sen](#page-16-2) [34] exploited the use of shared-memory multiprocessor computer architectures as well as data and task partition parallel programming models. Rehman et al. [2] developed a parallel approach of non-rigid registration by addressing it as an Optimal Mass Transport problem. Lapeer et al. [36] presented a point [base](#page-19-0)d registration method, integrating a Radial Basis Function (RBF) as a smoothing function and sought to mimic the interacting deformation of biological tissues. Mafi and Sirouspour [37] exploited a ²⁷⁷ GPU based computational platform for real-time analysis of soft object deformation. Ellingwood et al. [16] developed a [new](#page-17-5) computation- and memory-efficient Diffeomorphic Multi-Level B-Spline Transform Composite method on GPU for the non-rigid mass-preserving registration of CT volumetric images. The Sum of Squared Tissue Volume Difference (SSTVD) was adopted as the similarity criterion to preserve the computed tissue volume. A cubic B-Spline based FFD transformation model was used to capture the non-rigid deformation of objects like human lungs. The experiments used lung CT images, indicating an increase of speed of 112 times relative to the single-threaded CPU version, and of 11 times compared $_{285}$ to the 12-threaded version when considering the average time per iteration using the 286

GPU based implementation.

3 Material and Methods 288

As described in Section 2.1, non-rigid image [regist](#page-3-1)ration involves transforming $_{288}$ different sets of data into one coordinate system. Besides the optimization of an 290 objective function, its evaluation as well as the transformation of the floating image ²⁹¹ using splines and an interpolation function can be taken into account to accelerate 292 the FFD algorithm. In this study, the acceleration possibilities for the optimization $_{293}$ step were identified by realizing parallelization options using profiling ²⁹⁴ tools $[17, 18, 24]$ $[17, 18, 24]$ $[17, 18, 24]$.

3.1 Environment settings 296

The used test infrastructure included a desktop computer, with 16 GB of RAM $_{297}$ (DDR3-1600 MHz), an Intel(R) Core(TM) $17-4790$ 3.60 GHz processor, the Linux $_{298}$ Debian 8 operating system, the GNU gcd/g++ compiler 4.9.2, the Open 299 Multi-Processing (OpenMP) 3.1 application programming interface, Visual Studio 300 Code 1.57.1, gprof 2.25, perf 3.16.7-ckt20, gprof2dot $^{-1}$, and dot 2 2.38. 301 The used processor has four physical cores, and two logical threads can be 302 simultaneously run in each core using the support of a feature commonly known as 303 *hyper-threading technology*. Thus, one can choose to effectively run from a single 304 thread to a maximum of 8 threads depending on partial or complete utilization of the $\frac{305}{200}$ available cores. $\frac{306}{400}$

For the experiments, this study used Multiple Sclerosis (MS) images that were 307 collected from the MS Longitudinal Challenge Data Set repository [38], and are ³⁰⁸ freely distributed for research purposes. We randomly selected thirteen images from $\frac{308}{200}$ the original dataset to validate the non-rigid image registration results. All included 310 images were obtained using the same imaging scan and under the same acquisition $\frac{311}{211}$ conditions, i.e., using a 3.0 Tesla MR imaging scanner (Philips Medical System, ³¹² Best, The Netherlands), according to the following image acquisition parameters: $\frac{313}{2}$ *T*₁-weighted (*T*₁ − *w*) magnetization prepared rapid gradient echo (MPRAGE) with $TR=10.3$ ms, TE=6 ms, flip angle=8°, and 0.82x0.82x1.17 mm³ voxel size; a double TR=10.3 ms, TE=6 ms, flip angle= 8° , and $0.82 \times 0.82 \times 1.17$ mm³ voxel size; a double spin echo (DSE), which produces PD-w and $T_2 - w$ images with TR=4177 ms, 316 TE₁=12.31 ms, TE₂=80 ms, and 0.82×0.82×2.2 mm³ voxel size; and a $T_2 - w$ 317 fluid-attenuated inversion recovery (FLAIR) with TI=835 ms, TE=68 ms, and 318 $0.82x0.82x2.2$ mm³ voxel size [38].

¹ gprof2dot is an open source script written in Python used to convert the output from a range of profiles into a dot graph. This script can be freely downloaded at [https://github.com/](https://github.com/jrfonseca/gprof2dot) jrfonseca/gprof2dot.

² dot is a Graphviz feature for producing hierarchical drawings of directed graphs. Graphviz is an open source visualization software for representing structural information such as diagrams of abstract graphs. More information is available at http://graphviz.org.

3.2 Registration evaluation

 Dice Similarity Coefficient (DSC) is a simple and useful statistical validation metric commonly used to evaluate the performance of both registration reproducibility and spatial overlap accuracy against to registration ground truths [11]. The DSC value rates the overlap of two masks between 0 (zero) and 1 (one), where 1 (one) indicates a perfect overlap and 0 (zero) none. Therefore, DSC assesses the spatial overlap between the registration result (M_m) and the corresponding registration ground truth (M_p) as [11]:

$$
DSC = \frac{2||M_m \bigcap M_p||}{||M_p|| + ||M_p||},\tag{4}
$$

328 where $||M_m||$ and $||M_p||$ are the number of pixels, or voxels in 3D, in M_m and M_p , respectively. M_m is the area, or volume, of the registration obtained by the automated 330 algorithm whereas M_p is the area, or volume, of the registration ground truth and 331 *M_p* $\bigcap M_p$ is the overlapping area, or volume, of the two images under comparison.

3.3 Performance evaluation

 In order to estimate the speedup of the studied FFD algorithm, Amdahl's law of speedup was used [39]:

$$
Speedup_{enhanced} = \frac{1}{(1-f) + \frac{f}{S}},\tag{5}
$$

 where *Speedupenhanced* is the overall speedup of the algorithm, *f* the execution time of a function eligible for optimization, and S the expected speedup of this function. The key idea of this formula is to determine time-consuming functions in an implementation that can be adapted for optimization. Such functions (complete or partial) are frequently referred as bottlenecks. To gain significant overall speedup, $\frac{340}{40}$ the value of f should be high [29, 40, 30]. Once the [bottle](#page-18-7)[necks](#page-19-1) [are](#page-18-8) identified, possible optimizations are postulated to help the performance improvement. These optimizations should then be individually verified to ensure that they result in real measurable improvements.

The performance of the studied FFD algorithm was, therefore, improved concerning the bottlenecks that were perviously identified by using the profiling tools gprof and perf. We selected these tools as they combined profiling methods based on distinct approaches, i.e., those based on instrumentation, statistics, and event based. Each tool comprises two essential components. One is a runtime routine; profiling tool calls this routine at the beginning of every function that needs to be compiled with profiling parameters. The other component is the post-processing version of the algorithm under analysis that aggregates and presents the data. We compiled the FFD single thread based algorithm implementation with the following parameters: $(-fno-omit-frame-pointer)$ in order to enable the frame pointer analysis; $(-q)$ for generating symbol information, which in turn enabled source code analysis; and the parameter $-pq$, which is used for inserting the monitor function mcount before each function call. 356

The profiling tool maintains a careful calculation of the effective computation 357 times in different execution scenarios. The monitor function *mcount* records the function address and identifies the source of the cycles based on the addresses 355 generated inside the profiled function. When a child function is a member of a cycle, 360 the time shown is the appropriate fraction of the time for the complete cycle. 361 Self-recursive routines have their calls broken down into calls from the outside and 362 self-recursive calls; thus, only the outside calls affect the propagation time. $\frac{363}{100}$

gprof is relatively easy to employ and is portable though limited in scope. It ³⁶⁴ produces a detailed call graph identifying functions that call other functions and the 365 number of times each function is invoked. Furthermore, $qpp\sigma f$ lists the percentage 366 of time spent in a function and, hence, computes the execution time of that function. ³⁶⁷ perf uses statistical sampling to collect profile data thereby generating an ³⁶⁸ interruption at regular time intervals. perf can identify all processes running on the 16 CPU enabling it to capture all relevant information such as the program counter, and ³⁷⁰ the CPU core. Next, it writes all of this data to an output file called $perf$. data. 371 q prof and perf runtime routines gather accurate call counts that combined with a 372 post-processing version of the algorithm leads to an evaluation table that lists the 373 number of calls to each function, as well as the percentage amount of time spent in 374 such function, and the average time per call. 375

4 Results and Discussion 376 September 2018 12:36 September 2018 1376 376 September 2018

In this section, results from experiments performed aiming at getting useful profiling information, accumulating samples, and producing statistically meaningful results $\frac{378}{276}$ of the FFD algorithm under study using images of the MS Longitudinal Challenge dataset, are reported and discussed.

4.1 Algorithm evaluation 381

The implementation was profiled using 13 images, and then the accuracy was 382 evaluated by comparing the registration results with those obtained using a *classical* 383 FFD im[p](#page-10-0)lementation 3 . The obtained comparative results are presented in Table 1. \qquad 384

4.2 Computation time evaluation 385

The required runtime was investigated in order to evaluate the impact of the profile 386 based implementation in terms of computer performance. Each experiment was 387 executed fifty times for each image. Mean and standard deviation values of the time 388 required to process the profiled based algorithm were calculated. All input images 388 ³⁹⁰ were treated in the same manner, and the considered processing time includes the

³ An executable version of the FFD algorithm used for comparison purpose, by performing quantitative analysis based on the DSC value, can be downloaded from Daniel Rueckert's webpage: [http://www.](http://www.doc.ic.ac.uk/~dr) doc.ic.ac.uk/~dr

Image #	Dimension	DSC	
	256x256x35	0.97262	
\overline{c}	256x256x120	0.95407	
3	256x256x70	0.96194	
4	256x256x70	0.96071	
5	256x256x70	0.97167	
6	256x256x120	0.93503	
7	256x256x70	0.94767	
8	256x256x70	0.95950	
9	256x256x70	0.96650	
10	256x256x120	0.97314	
11	256x256x70	0.96029	
12	256x256x70	0.95365	
13	256x256x120	0.96493	

Table 1 Comparison of *classical* and profiled based FFD algorithms: results for 13 images based on the DSC value.

Table 2 Means and standard deviations of the runtime (in seconds) required by the profiled based FFD algorithm implementation.

Image #	Dimension	Runtime
1	256x256x35	73.08411 ± 0.05945
$\overline{\mathcal{L}}$	256x256x120	79.00041 ± 0.07101
$\overline{\mathcal{E}}$	256x256x70	74.08051 ± 0.01961
$\overline{4}$	256x256x70	73.48618 ± 0.01920
5	256x256x70	$74.20270 + 0.01393$
6	256x256x120	79.00217 ± 0.07294
7	256x256x70	$74.68039 + 0.01279$
8	256x256x70	74.99707 \pm 0.01484
9	256x256x70	73.94009 ± 0.01752
10	256x256x120	79.07080 ± 0.07590
$\overline{11}$	256x256x70	74.08260 ± 0.01220
12	256x256x70	74.83009 ± 0.01522
13	256x256x120	79.00239 ± 0.08677

 time spent to load the data into the main system memory until the end of the registration process, i.e., when the resultant image has been produced. The obtained

results are presented in Table 2.

4.3 Performance analysis

 Concerning the performance analysis, it should be noted that the profiling tools collect data while monitoring performance counters, hardware interruptions, and operating system calls. Profiling tools periodically interrupt the kernel of the operating system to record a new sample; these samples are stored in a ring buffer, generating, therefore, overhead. perf mitigates sampling overhead by enforcing local sampling buffer. perf creates one instance of the event on each used CPU; then, the events are effectively measured when that CPU executes each thread. All the samples are aggregated into a single output file once all profiles have been run. In the experiments, the sampling mode in perf was used to trace the events of the FFD algorithm in real-time. For the experiments conducted with 2, 4, and 8 threads, 404 perf generated output files with sizes of dozens of megabytes. The obtained results are presented in Table 3. This considerable [bi](#page-12-0)g data size is because $perf$ depends 406 on the adopted frequency. For the sampling rate according to the events are 407 recorded, a rate of 4000 samples per second was used, which resulted in high 408 overhead and large output files. However, *gprof* generates output files that are in the ⁴⁰⁹ order of kilobytes, i.e., 768 KB; mainly, because the output file contains a histogram ⁴¹⁰ of program counter samples and the arc table. ⁴¹¹

Image #	Dimension	Number of threads			
		1 Thread	2 Threads	4 Threads	8 Threads
	256x256x35	9.65	10.23	12.24	25.41
$\overline{2}$	256x256x120	12.40	13.16	18.91	28.74
$\overline{\mathcal{E}}$	256x256x70	11.08	11.83	13.02	27.53
$\overline{4}$	256x256x70	17.74	18.48	22.31	25.42
- 5	256x256x70	36.32	32.37	61.61	25.45
6	256x256x120	8.61	8.92	9.84	25.63
	256x256x70	9.16	9.45	25.53	33.02
8	256x256x70	7.85	8.23	11.33	30.56
9	256x256x70	12.04	12.51	15.92	30.22
10	256x256x120	24.07	25.04	31.32	50.58
11	256x256x70	18.44	19.14	25.81	52.37
12	256x256x70	12.91	13.60	14.64	31.66
13	256x256x120	20.51	21.20	33.58	46.05

Table 3 File sizes, indicated in megabytes (MB), generated by perf according to the dimension of the input images and the developed OpenMP based implementation using different number of threads.

In order to extract performance statistics and record the arcs in the call graph, 412 the collected data was analyzed. The call graph represents information intuitively by 413 employing a visual map from a collection of hierarchical data in order to quickly 414 facilitate its understanding $[41, 42, 43]$. The call graph [re](#page-19-2)[prese](#page-19-3)[nts](#page-19-4) time consuming 415 functions and the number of times the functions were invoked. By analyzing the ⁴¹⁶ call graph sample of the image registration algorithm, the graph shown in Fig. $2 \text{ was } 417$ generated, which includes the time propagated for each function from its descendants, ⁴¹⁸ and the number of times each function was called. 418

The built call graph displays the descendants as well as the caller of each function, including the time propagated to each routine from its descendants. The important entries of the call graph profile are the ones depicted with grey numbered circles in Fig. 2, which ref[ers](#page-13-0) to the primary function of the studied non-rigid image registration algorithm. In this figure, *element* 2 represents the name of the caller function; the percentage of the runtime accounted by the algorithm's function and its $_{425}$ descendants is indicated by *element 3*; *element 4* accounts for a time that depends on ⁴²⁶ whether it is the primary function for that section, the function's caller or descendant functions. In the first case, time is the actual function execution time during the running of the algorithm. In the second case, it indicates the amount of the self-time being propagated to that caller, based on the percentage of calls to the primary 430 function made by that caller. Finally, for descendant functions, it represents the 431

Fig. 2 Call graph generated by perf representing the most often called functions in the studied image registration algorithm.

 amount of that descendant function's self-time being propagated to the primary function based on the percentage of calls made to the descendant by the primary function. The last entry in the green box next to *element 4* represents the number of times that function was called (63x in this case), and finally *element 5* is related to the accumulated percentage of time running a function, taking into account the propagation for each descendant function.

 The built call graph is helpful in evaluating the algorithm's performance and identifying its bottlenecks. Taking full advantage of profiling tools requires to focus on the analysis of the relevant parts of the algorithm execution, making the experiments easier to understand. The used profiling tools identified that the 442 function reg getEntropies is responsible for 68% of the total running time (56*.*90 seconds), meaning that joint histogram filling is the main time consuming task within this function. The other costly functions identified by the profiling tools were:

 – reg_cubic_spline_getDeformationField3D which generates the deformation field: a lattice of equally spaced control points is defined over the reference image using cubic B-splines;

 449 – ResampleImage3D which computes the value $I_e(T(x))$ for every pixel *x*, or voxel in 3D, inside the reference image. In this case, the computational complexity is linearly dependent on the number of pixels, or voxels, in the reference image;

 – UpdateParameters which assesses the quality of a registration using a cost function such as mutual information. In order to achieve the perfect registration between two images, the transformation parameters are iteratively optimized.

 To obtain a measure of the runtime, the program that implements the algorithm was run a total of fifty times. The average of the time elapsed reported by each profiling tool was calculated. In all cases, the execution times for different runs of

the implementation were remarkably consistent. The most time consuming functions 458 iterate a hundred times which makes them desirable parallelization targets. The algorithm under study requires large computations, iterating until convergence, ⁴⁶¹ aiming to ensure the best possible registration of the input images. Hence, the 462 studied algorithm can greatly benefit from a high degree of parallelism.

For the performance analysis of the developed parallel implementation, a 464 benchmark problem was defined, which is suitable to evaluate the performance in ⁴⁶⁵ the setting for sequential execution as well as parallel execution. Next, the effect of $_{466}$ using different number of physical cores on the performance of the multi-threaded 467 algorithm was studied. For a fixed number of cores, the same number of threads per ⁴⁶⁸ core for execution, i.e., one thread for each core, was used. The costly function 468 reg_getEntropies was implemented using OpenMP. 470

All the experiments that were previously carried out were repeated. The results 471 were then compared using varying degrees of parallelism, in particular using 1, 2, 4 472 and 8 threads. As shown in Fig. 3, the dev[el](#page-14-0)oped parallel OpenMP based 473 implementation achieved a significant reduction in the runtime of the studied 474 non-rigid image registration algorithm relatively to its single-thread implementation. ⁴⁷⁵ This validates the claim that profiling tools could help programmers to quickly ⁴⁷⁶ identify critical bottlenecks.

Fig. 4 [dep](#page-15-1)icts the performance gain of the parallel OpenMP based 478 implementation of the non-rigid image registration algorithm. One can see that it 479 scales almost exponentially, and that the parallel implementation achieved runtime 480 ⁴⁸¹ approximately seven times faster than those of the single thread based ⁴⁸² implementation, taking into account the ratio of the sequential runtime to the ⁴⁸³ parallel runtime. The parallel based implementation uses one thread for each core

⁴⁸⁴ for the execution, which is created and managed by the multi-threading library.

using the developed OpenMP based implementation of the FFD algorithm.

Fig. 4 Means and standard deviations of runtime spent for running the developed OpenMP based implementation of the FFD algorithm under study.

⁴⁸⁵ 5 Conclusion and Future Work

 The need for parallelization is on the rise, in part due to the facts that many computing devices now have multi-core processors available, and the applications are becoming more complex and have to deal with larger amounts of data. However, writing parallel code is a challenging task for many programmers, since it involves a strong learning curve for coding applications in parallel design, and requires a strong understanding of advanced concepts concerning memory hierarchy and optimal data paths in computer systems. While much effort has been devoted to address the issue of parallel programming, the current work was mainly focused on gathering parallelization support from profiling tools. As our findings suggest, profiling tools can be highly effective to detect and evaluate performance bottleneck snippets in a non-rigid image registration algorithm based on FFD, providing a low-impact method for gathering useful information.

 The developed parallel OpenMP based implementation was compared against the corresponding single thread based implementation in several experiments. The parallelization of the costly functions of the FFD algorithm reduced the runtime by up to 7 times compared to the single thread version. 501

In conclusion, the proposed parallelization based on profiling tools substantially 502 improved the runtime performance of the studied non-rigid image registration $\frac{503}{20}$

algorithm. This will facilitate medical practitioners and researchers that commonly 504 rely on image registration to label anatomical data, identify diseases, compare 505 patient images and perform follow-up diagnosis. This is, therefore, a step forward to $_{506}$ make accelerated non-rigid image registration solutions more accessible to a broader $_{507}$ audience.

In future work, we will develop the approach further to address the all time $\frac{500}{200}$ consuming functions already detected in this study, with an objective to make these $\frac{1}{510}$ functions more efficient so that better speedups can be possible with more modern 511 data parallel algorithms. We plan to optimize the performance of these algorithms 512 by using heterogeneous parallel computing platforms that use GPUs. Additional $\frac{513}{2}$ challenges need to be addressed, for instance, the issue in shared memory systems of $_{514}$ protecting simultaneous data access in order to avoid data inconsistency and errors, ⁵¹⁵ load balancing, and the efficient management of reading/writing data to massive $\frac{1}{516}$ data units. These challenging elements are all critical for achieving efficiency and 517 the maximum performance possible in the underlying architecture. Finally, another $\frac{518}{2}$ interest future work would be the comparison of the suggested approach against 519 related methods available in the literature. 520

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