USING PHYSICAL PRINCIPLES IN THE SEGMENTATION OF OBJECTS REPRESENTED IN IMAGES

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1. ABSTRACT

The goal of this work is to automatically extract the contour of an object represented in an image after manually defining an initial contour for it. This rough initial contour will then evolve until it equals the border of the desired object. The contour is modelled through a physical formulation, using the Finite Element Method, and its evolution to the desired final contour of the object to segment is governed by: internal forces, defined by the intrinsic physical characteristics selected for the model; and external forces, defined in function of the image features most suitable for the desired object. To build the physical model, an isoparametric finite element is employed and to obtain its evolution towards the object border the dynamic equilibrium equation is solved.

2. INTRODUCTION

In the domain of Computational Vision, the identification of an object represented in an image, usually designated by segmentation, is one of the most common and complex tasks. Usually, whenever it is intended to extract higher-level information from an image or even from image sequences, the used process starts by segmenting the input image(s). Thus, image segmentation is one of the working areas in Computational Vision with more research done and so it will probably be throughout the times.
The main goal of our work is to extract the contour of an object represented in an image, after the definition of an initial contour for it. This contour, roughly defined by the user of the developed implementation, will evolve throughout an iterative process until it reaches the border of the desired object. For that purpose, it was decided to use deformable models defining their behaviour according to physical principles, as proposed by [1]. Thus, the dynamic equilibrium equation is used, also known as equation of motion, and applied to the elastic physical model built using the Finite Elements Method.
The here used methodology is briefly described in Fig. 1. Its first step consists in drawing a contour on the input image that is close to the border of the object to segment. That shape is considered as the initial segmentation contour for that object. Next, this contour is modelled according to physical principles using the Finite Elements Method; namely, by adopting Sclaroff's isoparametric finite element [2]. To move the model to the border of the object, the dynamic equilibrium equation, that describes the equilibrium between the internal and external forces involved, is solved. The internal forces are defined by the physical characteristics adopted for the model, determined by the virtual

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material used and the interaction between its nodes; and the external forces are defined in terms of the image features that best describe the object.

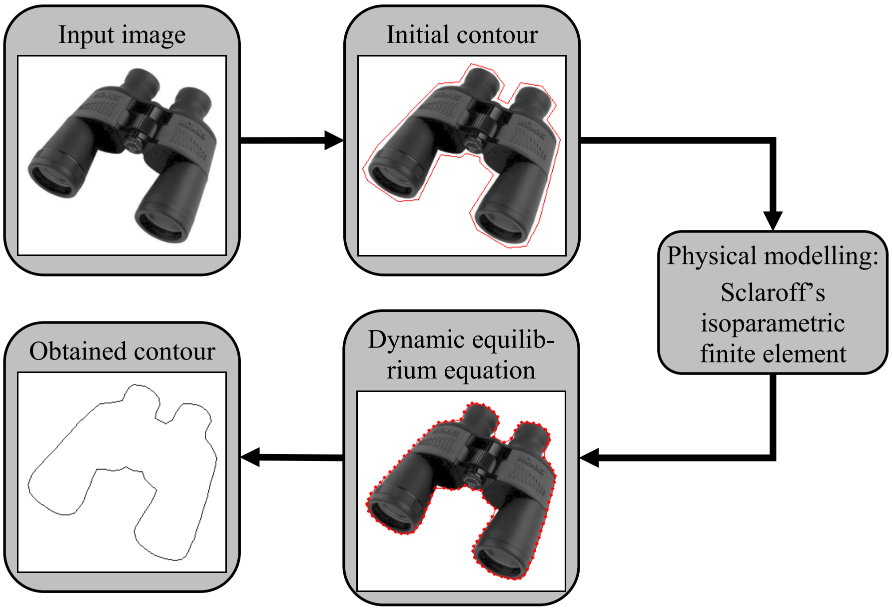

Fig. 1: Schema of the methodology used to segmented objects represented in images.

3. PHYSICAL MODELLING

After defining the initial contour for the object, we model it in physical terms; that is, we assign mass, stiffness and damp to each point of the contour, that is, to each node of the model used.

To model the initial contour and simulate its elastic behaviour, [1] used affine interpolation functions together with finite differences. In this work, we employ the Finite Element Method and Gaussian interpolants instead. Namely, we use Sclaroff's isoparametric finite element, [2], that uses a set of radial basis functions that allows an easy insertion of the data points in the model. With this isoparametric finite element, when an object is modelled it is as if each of its feature points are covered by an elastic membrane [3; 4].

For the m points $\mathbf{X}_i(x_i, y_i)$ of the initial contour, the mass matrix of Sclaroff's isoparametric element, $\mathbf{M}$, is defined as, [2-4]:

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}' & 0 \\ 0 & \mathbf{M}' \end{bmatrix}, \quad (1)$$

where $\mathbf{M}'$ is a sub-matrix $m \times m$ defined as $\mathbf{M}' = \rho \pi \sigma^2 \mathbf{G}^{-1} \mathbf{\Gamma} \mathbf{G}^{-1}$, where ρ is the mass density of the virtual material adopted, and the elements of matrix $\mathbf{\Gamma}$ are the square roots of the elements of matrix $\mathbf{G}$ defined as:

$$\mathbf{G} = \begin{bmatrix} g_1(\mathbf{X}_1) & \cdots & g_1(\mathbf{X}_m) \\ \vdots & \ddots & \vdots \\ g_m(\mathbf{X}_1) & \cdots & g_m(\mathbf{X}_m) \end{bmatrix}. \quad (2)$$

with:

$$g_i(\mathbf{X}_j) = e^{-\left\| \mathbf{X}_j - \mathbf{X}_i \right\|^2 / 2\sigma^2}, \quad (3)$$

where σ is the standard deviation that controls the interaction between the model nodes. On the other hand, the stiffness matrix, **K**, is given by:

$$\mathbf{K} = \begin{bmatrix} \mathbf{K}_{11} & \mathbf{K}_{12} \\ \mathbf{K}_{21} & \mathbf{K}_{22} \end{bmatrix}, \tag{4}$$

where $\mathbf{K}_{ij}$ are symmetric $m \times m$ sub-matrices depending on constants that are functions of the virtual material adopted for the contour, [2; 3; 5].
Finally, we use Rayleigh's damping matrix, **C**, which is a linear combination of the mass and stiffness matrices with constraints, μ and γ, based upon the chosen critical damping, [6; 7]:

$$\mathbf{C} = \mu\mathbf{M} + \gamma\mathbf{K}. \tag{5}$$

4. EQUILIBRIUM EQUATION

After having the initial contour transformed into an elastic physical model we need to estimate its evolution in the direction of the object edges to achieve the desired segmentation. To achieve this goal, the second order ordinary differential equation, commonly known as Lagrange's dynamic equilibrium equation, is solved:

$$\mathbf{M}\ddot{\mathbf{U}}^t + \mathbf{C}\dot{\mathbf{U}}^t + \mathbf{K}\mathbf{U}^t = \mathbf{F}^t, \tag{6}$$

for each time step t, where $\mathbf{U}$, $\dot{\mathbf{U}}$ and $\ddot{\mathbf{U}}$ are, respectively, the displacement, velocity and acceleration vectors, and **F** represents the external forces, [8]. This equation describes the equilibrium between the internal and external forces involved on the model nodes. The internal forces are defined by the physical characteristics adopted for the model, determined by the adopted virtual material and the level chosen for the interaction between the nodes of the model, which is considered while building Sclaroff's isoparametric finite element. The external forces, **F**, are determined by the image features that best describe the object to segment. In particular, the intensity value of each pixel of the initial image, $\mathbf{F}_{int}$, the value of the pixels of the edges image, $\mathbf{F}_{edg}$, and the distance from each pixel to the nearest edge, $\mathbf{F}_{dist}$:

$$\mathbf{F} = \mathbf{F}_{edg} + \mathbf{F}_{int} + \mathbf{F}_{dist}. \tag{7}$$

Here, the edges image is obtained by applying Shen & Castan's edge detection operator, [9], to the original image, and the distance image is obtained by calculating the distance of each pixel to its nearest edge using Chamfer's method.
After the physical modelling of the initial contour, our algorithm calculates the line orthogonal to the line tangent to the contour at each node of the model. It is along each one of these lines that the forces are calculated. Denoting as Q_i all the pixels belonging to the orthogonal line of node P, the edges force at point P is given by:

$$\mathbf{F}_{edg}(P) = k \frac{\sum_{i=1}^{N} Edg(Q_i)}{N}, \tag{8}$$

where $Edg(Q_i)$ is the value of the pixel Q_i in the edges image, k is a stiffness constant and N is the number of pixels of the orthogonal line. The intensity and distance forces equations are similar to Eq. (8).
If the mean of the edges values of the N pixels of the orthogonal line is lesser than a given value, *val*, then the line will continually grow until the mean reaches *val*. Thus,

each line has the length it needs to determine a sufficient force to move its associated node.

5. SOME RESULTS

To illustrate the results of the methodology considered to segment an object represented in an image by identifying its contour, consider the images in Fig. 2. In the first one we can see, in red, the initial contour manually defined for the object. The second image represents the segmentation obtained using a physical model with 122 nodes (data points) made of rubber, and considering k=200N/m. In this case, the computational process took 56s to achieve the final result. (In this work we used a personal computer with an Intel Pentium D at 3GHz processor and 2GB of RAM.)

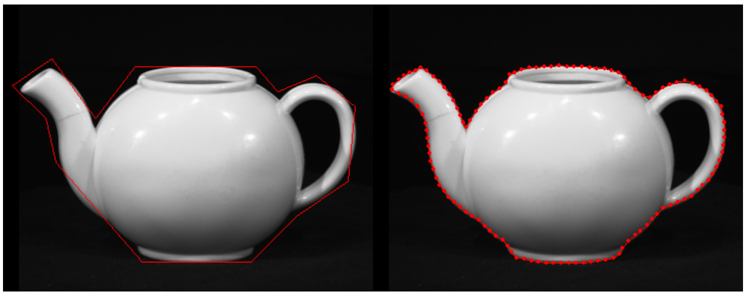

Fig. 2: Initial contour (left); result of the segmentation process using k=200N/m and considering a rubber model (right).

If the same initial contour is modelled with copper instead of rubber, using k=200N/m would not practically move the contour. Because copper is more rigid than rubber it can support bigger external forces, so if k takes bigger values the process continues to run without numerically diverging. In fact, using $k=1\text{x}10^6$N/m in the 122 nodes model made of copper, the segmentation is achieved after 5 minutes. Even with a much higher k the segmentation process using a copper model takes a lot longer to finish than using rubber as the adopted virtual material, which is consistent with the expected behaviour of real objects, because it is easier and faster to deform objects made of rubber than of copper.
The initial contour in Fig. 2 was drawn close to the object to segment; however, because of the adaptive approach considered for the external forces, that does not have to be. The initial contour can be drawn further away from the object border, but that slows down the segmentation process because each pixel has a longer path to go through. The example in Fig. 3 uses the same object and the same initial parameters to build the physical model as the one in Fig. 2 but with an initial contour defined further away from the object to segment. In this case, the segmentation process takes 178s to finish.

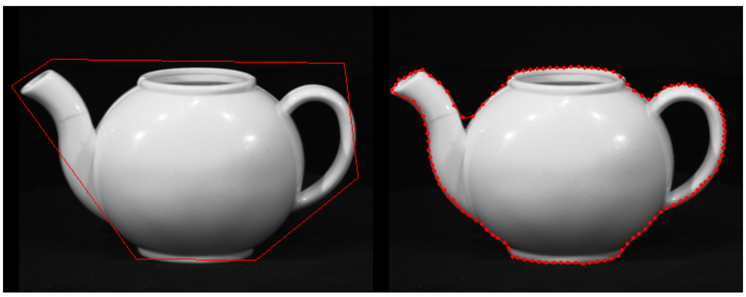

Fig. 3: Initial contour defined further away from the object (left); result of the segmentation process using k=200N/m and a rubber model (right).

In the previous example no nodes redistribution was made, i.e., the nodes did not keep the distance between them along the segmentation process. That is why areas with high curvature may not be well segmented, as is the case here. The final contour shown in Fig. 3 presents a small number of points in the area between the beak and the opening of the teapot, preventing the junction of the beak and the body of the teapot to be well represented. By forcing the nodes to maintain a given distance between them, the number of nodes increases or decreases along the segmentation process in order to obtain a more accurate result, Fig. 4.

Fig. 4: Result of the segmentation process with nodes redistribution.

In the case of more complex segmentation cases, such as when the objects are overlapped, like the ones in Fig. 5, the final result may not be the expected one, because features belonging to other objects can be stronger than the ones of the object to segment, consequentially attracting some nodes of the model to the wrong object.

Fig. 5: Initial contour (left); result of the segmentation with k=2,000N/m considering a model with 50 nodes and made of rubber (right).

6. CONCLUSIONS AND FUTURE WORK

In this paper, a methodology to segment objects represented in images based on physical principles was presented.
The experimental results obtained using our physically driven segmentation methodology, some presented in this paper, are quite satisfactory. However, our approach has two major problems:
1) It becomes slower as the number of nodes of the model used in the segmentation process increases; what can be very inconvenient when a detailed contour extraction needs to be accomplished and the application demands very fast results.
2) The segmentation result can be compromised when the image in which the object is represented is very complex, with noisy data or objects overlapped, for instance.
Because of these two major problems, in the near future some changes to fasten and im-

prove the segmentation process will be introduced, such as trying different approaches for the definition of the external forces, and the development of computational parallel implementations. The use of finite elements suitable for large deformations is also a subject to be addressed in the following stages of this work.
The segmentation of some specific type of images will be also considered. For that, it will become necessary and advantageous the introduction of adequate constraints in the segmentation methodology to improve its robustness.

7. ACKNOWLEDGMENTS

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